ME303: Assignment 2

Due: 27/3/23

All questions in this assignment are related to a circular domain of radius a.

1. If $Q = 0$, and if the boundary condition is $T|_{r=a} = g(\theta)$, then find the steady-state solution $T(r, \theta)$. As a special case, if $g(\theta) = T_0$, where T_0 is a constant, then determine $T(r, \theta)$ from your general solution.

2. If $Q(r, \theta) \neq 0$, and the boundary condition is given by $T|_{r=a} = 0$, then set out the mathematical procedure for determining $T(r, \theta)$. As a special case, if $Q = Q_0$, where Q_0 is a constant, find an expression for $kT(r, \theta)/(\rho a^2 Q_0)$ (you need not simplify any infinite sums).

3. The remaining problems are radially symmetric (no dependence on θ)

   (a) If $T|_{r=a} = T_a(t)$, then by using the Laplace transform method, find an expression for $T(r, t)$ in terms of $T_a(t)$. If $T_a(t) = T_0$, where T_0 is a constant, find $T(r, t)$ from your general solution. By taking the limit as $t \to \infty$, determine if this solution reduces to the steady-state solution in Question 1 above.

   (b) If $-k(\partial T/\partial r)_{r=a} = q_a(t)$, then find an expression for $T(r, t)$ in terms of $q_a(t)$. If $q_a(t) = q_0$, where q_0 is a constant, find $T(r, t)$ from your general solution. Does a steady-state solution exist for this boundary condition?

   (c) If $T_a(t) = Q = 0$, and the initial temperature is $f(r)$, then determine the evolution of the temperature field $T(r, t)$. Find $T(r, t)$ for the special case where $f(r) = C_1$, where C_1 is a constant.

   (d) If $f(r) = T_a = 0$, and $Q(r, t)$ is nonzero, then set out a mathematical procedure for finding $T(r, t)$. For the special case when $Q(r, t)$ is a function of time alone, i.e., $Q(r, t) \equiv Q(t)$, find an expression for $T(r, t)$. Further, if $Q(t) = Q_0$, find $kT/(\rho a^2 Q_0)$. By taking the limit as $t \to \infty$, determine if this solution reduces to the steady-state solution in Question 2 above.