ME303: Assignment 3

Due: 13/4/23

All questions in this assignment are related to the region inside a circular domain of radius r_1.

1. If the normal acceleration is prescribed at $r = r_1$ as $A(t) = Vi\omega e^{i\omega t}$, then find the periodic steady-state solution for the pressure.

2. If instead of the normal acceleration, the pressure is prescribed at $r = r_1$ as $P(t) = P_0 e^{i\omega t}$, then find the periodic steady-state solution for the pressure.

3. If the normal acceleration $a_r|_{r=r_1}$ is prescribed as $A(t)$, then find the transient solution for the pressure field. As special cases, find the solution for $A(t) = A_0$ and $A(t) = A_0 \sin \omega t$.

4. If the surface $r = r_1$ is rigid, and the initial conditions are $p_\Delta(r, 0) = p_0(r)$ and $\dot{p}_\Delta(r, 0) = v_0$, then find the evolution of the pressure field, i.e., $p_\Delta(r, t)$. As a special case, if $p_0(r) = p_0$, where p_0 is a constant, find $p_\Delta(r, t)$.

5. If the boundary condition is $p_\Delta|_{r=r_1} = 0$, and the initial conditions are $p_\Delta(r, 0) = p_0(r)$ and $\dot{p}_\Delta(r, 0) = v_0$, then find the evolution of the pressure field, i.e., $p_\Delta(r, t)$. As a special case, if $p_0(r) = p_0(1 - \xi^2)$, where $\xi = r/r_1$, find $p_\Delta(r, t)$.

6. If $\tilde{p}|_{t=0} = \dot{\tilde{p}}\big|_{t=0} = \tilde{p}|_{r=r_1} = 0$, then find the solution to the nonhomogeneous equation

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \tilde{p}}{\partial r}\right) + G(r, t) = \frac{1}{a_0^2}\frac{\partial^2 \tilde{p}}{\partial t^2}.$$

   As a special case, if $G = G_0\psi(t)$ over the circular region $0 \le r \le r_2$ where $r_2 \le r_1$, and zero elsewhere, find the solution $\tilde{p}(r, t)$. Due to its analogy with membranes, this special case solves the problem of the vibration of a circular membrane fixed at its boundary, and loaded on its top surface.