Indian Institute of Science
ME 303: Endsemester Exam

Date: 19/4/2023.
Duration: 9.30 a.m.–12.30 a.m.
Maximum Marks: 100

1. This problem can be treated as a two-dimensional steady-state problem. Consider a circular inclusion of radius R, and with conductivity k_1 embedded in an unbounded domain with conductivity k_2. The far-field temperature is given by V_0+V_1xy, i.e., $\lim_{r\to\infty} T = V_0+V_1xy$, where r denotes the radial coordinate in a polar coordinate system, and $y = r\sin\theta$. Find the steady-state temperature distribution within the inclusion and in the surrounding region. Your solution procedure should be systematic with *no guesswork allowed.* (20)

2. The governing equation for the velocity component $u_z(x, y)$ for Poiseuille flow through a conduit of rectangular cross section (see Fig. 1) is given by (25)

$$\frac{\partial^2 u_z}{\partial x^2} + \frac{\partial^2 u_z}{\partial y^2} = -\frac{G}{\mu},$$

where G and μ are constants. The boundary condition is the no-slip condition $u_z|_{x=\pm a/2} = u_z|_{y=\pm b/2} = 0$. Find the (steady-state) velocity distribution $u_z(x, y)$. Once again, the solution should be derived systematically with no guesswork allowed. *State all the assumptions that you make regarding the nature of the solution.*

3. An infinitely long circular cylinder of radius a (i.e., the domain is $[0, a] \times (-\infty, \infty)$ in the r-z plane) with insulated boundary $r = a$, is subjected to a heat input $Q(r, z, t)$. Derive the temperature distribution $T(r, z, t)$. You may also directly use the following result for $u = J_k(\lambda r), Y_k(\lambda r)$: (20)

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{du}{dr}\right) = -\left(\lambda^2 - \frac{k^2}{r^2}\right) u.$$

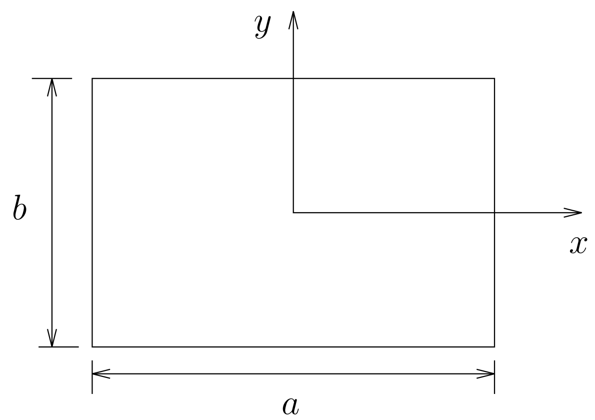

Figure 1: Flow through a rectangular duct.

4. The equation for torsional oscillations of a circular cylinder of radius a and length L is given by (35)

$$\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial (r u_\theta)}{\partial r}\right) + \frac{\partial^2 u_\theta}{\partial z^2} = \frac{1}{c^2}\frac{\partial^2 u_\theta}{\partial t^2}, \tag{1}$$

where c is the wave speed. With ρ denoting the density, the boundary conditions are $u_\theta\big|_{z=0} = 0$, and

$$\left.\frac{\partial u_\theta}{\partial z}\right|_{z=L} = \frac{rT(t)}{\rho c^2}.$$

Assume that the boundary conditions on the surface $r = a$ are automatically satisfied.

(a) By substituting $u_\theta(r, z, t) = rw(z, t)$, transform Eqn. (1) into a partial differential equation in terms of $w(z, t)$. Find the boundary conditions also in terms of $w(z, t)$.

(b) Assuming homogeneous initial conditions, find the solution for $w(z, t)$ in terms of $T(t)$.

(c) For the special case $T(t) = T_0$, where T_0 is a constant, find the solution (you need not evaluate summations that arise).

(d) By multiplying the governing equation for $w(z, t)$ that you have derived by an appropriate quantity, and integrating over the domain $[0, L]$, derive an equation of the form

$$\frac{dH}{dt} = \text{RHS}, \tag{2}$$

where

$$H = \int_0^L [\ldots]\, dz,$$

where the integrand in the above expression is in terms of w and its derivatives, and the RHS in Eqn. (2) comprises boundary terms involving $T(t)$. If $T(t)$ is suddenly set to zero at say time t_0, deduce a 'conservation law' for H from Eqn. (2) for $t \geq t_0$.