Indian Institute of Science
ME 303: Midsemester Test

Date: 25/2/2023.
Duration: 9.30 a.m.–11.00 a.m.
Maximum Marks: 100

1. The relation between the stream function ψ, which is a harmonic function, and the velocity components (u_r, u_θ) in a polar coordinate system is given by (60)

$$u_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta},$$
$$u_\theta = -\frac{\partial \psi}{\partial r}.$$

For a certain type of two-dimensional flow around a circular cylinder of radius R, it is given that the far-field velocity as $r \to \infty$ is $U\boldsymbol{e}_x$, where $\boldsymbol{e}_x$ is the x-direction basis vector in the Cartesian system. In addition, it is also given that for every circular contour of radius r,

$$\int_0^{2\pi} u_\theta (2\pi r)\, d\theta = \Gamma \quad \forall r,$$

where Γ is a constant. The boundary condition at the surface of the cylinder is $\psi|_{r=R} = 0$. *Starting from the most general separable form for* ψ, deduce the actual expression for ψ for the given problem. The constants in the (infinite series) solution should be systematically deduced, and no 'guesswork' is allowed. (Hint: The function ψ is *not* a physical variable, and it is just a mathematical function introduced for convenience.)

2. A sphere of radius a is subjected to a constant heat input $Q = Q_0$, and a constant normal heat flux $\boldsymbol{q} \cdot \boldsymbol{n}$ at the surface. For axisymmetric problems, the volume and surface elements are given by $dV = 2\pi r^2\, dr\, d\xi$ and $dS = 2\pi a^2\, d\xi$, respectively, with associated limits of integration for ξ being $[-1, 1]$. (40)

   (a) Find the constant normal flux $\boldsymbol{q} \cdot \boldsymbol{n}$ that must be imposed at the surface of the sphere in order that a steady-state solution exists.

   (b) Provide an explanation for why superposition of the solutions for a normal flux $\boldsymbol{q} \cdot \boldsymbol{n}$ with $Q = 0$, and a nonzero Q with $\boldsymbol{q} \cdot \boldsymbol{n} = 0$ on the surface, *cannot* be used to obtain the solution to this problem (Hint: what are the constraints on $\boldsymbol{q} \cdot \boldsymbol{n}$ when $Q = 0$).

   (c) After making suitable assumptions (which you should state clearly), find the solution for the temperature field (modulo a constant) without using superposition (Hint: governing equation).