Indian Institute of Science
ME 303: Midsemester Test 2

Date: 1/4/2023.
Duration: 9.30 a.m.–11.00 a.m.
Maximum Marks: 100

1. Consider the semicircular domain of radius a shown in Fig. 1. The edges $\theta = \pm\pi/2$ are insulated, i.e., $\boldsymbol{q} \cdot \boldsymbol{n} = 0$ on these edges, and $-k(\partial T/\partial r)_{r=a} = q(t)$. The initial conditions and the heat input Q are both zero. (45)

   (a) Take the Laplace transform of the governing equation

   $$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t},$$

   and write the general solution of the Laplace-transformed temperature $\bar{T}(r, \theta, s)$.

   (b) Using the boundary conditions, find the constants in the general solution for $\bar{T}$.

   (c) Invert the resulting Laplace transform, and find the solution for $T(r, \theta, t)$ in terms of $q_a(t)$.

   (d) If $q_a(t) = t(2 - t)$ for $0 \le t \le 2$, and zero thereafter, find $T(r, \theta, 1)$ and $T(r, \theta, 3)$ (you need not evaluate any integrals that arise).

2. Consider the domain to be the entire three-dimensional space (i.e., all of $\Re^3$). Let $Q = Q(t)$ for $r \le a$ and $Q = 0$ for $r \ge a$, where r denotes the spherical radial coordinate, and let the initial temperature be zero. After making appropriate assumptions about the nature of the solution, find the solution for the temperature field in the entire unbounded domain in terms of $Q(t)$. You may directly use any inversion results for Bessel/spherical Bessel functions from the notes. You may also directly use the following result for $u = j_k(\lambda r), y_k(\lambda r)$: (55)

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{du}{dr}\right) = -\left(\lambda^2 - \frac{k(k+1)}{r^2}\right)u.$$

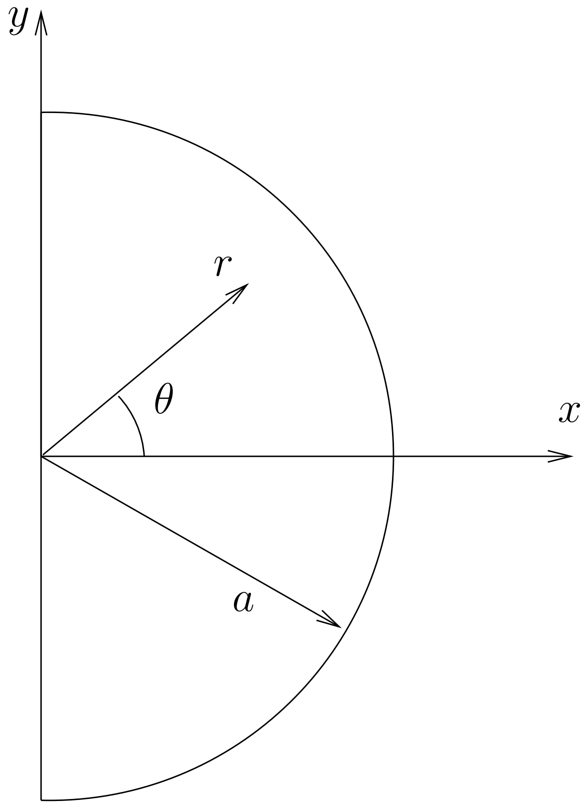

Figure 1: Semicicular domain.