ME242: Assignment 4

Due: 17/10/14

1. Show that for a linear isotropic material ($\boldsymbol{\tau} = \lambda(\operatorname{tr}\boldsymbol{\epsilon})\boldsymbol{I} + 2\mu\boldsymbol{\epsilon}$), the principal axes of strain coincide with the principal axes of stress. Hence, in this case, no distinction needs to be made between the principal axes of strain and the principal axes of stress, and one can refer to them simply as the principal axes.

2. We saw in the previous assignment that the stress distribution for a prismatic bar of length L hanging from a support is given by $\tau_{zz} = \rho g(L - z)$, other stress components zero. If the Young modulus of the material is E and Poisson ratio is ν, find the expressions for the strains. Verify that they satisfy the equations of compatibility. Show that the displacement field

$$u = -\frac{1}{E}\nu\rho g x(L - z), \quad v = -\frac{1}{E}\nu\rho g y(L - z), \quad w = \frac{1}{2E}\rho g\left(2zL - z^2 - \nu(x^2 + y^2)\right),$$

corresponds to the strains.

3. Show that the strain field $\epsilon_{xz} = \frac{1}{2}\left(\frac{\partial\phi}{\partial x} - y\right)$, $\epsilon_{yz} = \frac{1}{2}\left(\frac{\partial\phi}{\partial y} + x\right)$ other components zero, where $\phi = \phi(x, y)$, satisfies the equations of compatibility. By integrating the strain-displacement relations, determine the displacement field (Note: Do not forget to include displacements corresponding to rigid motion).

4. Considering the momentum balance of the element shown in Fig. 1, derive the following linear momentum equations (assume $\tau_{r\theta} = \tau_{\theta r}$)

$$\rho\frac{\partial^2 u_r}{\partial t^2} = \frac{\partial\tau_{rr}}{\partial r} + \frac{1}{r}\frac{\partial\tau_{r\theta}}{\partial\theta} + \frac{\tau_{rr} - \tau_{\theta\theta}}{r} + \rho b_r,$$
$$\rho\frac{\partial^2 u_\theta}{\partial t^2} = \frac{1}{r}\frac{\partial\tau_{\theta\theta}}{\partial\theta} + \frac{\partial\tau_{r\theta}}{\partial r} + \frac{2\tau_{r\theta}}{r} + \rho b_\theta.$$

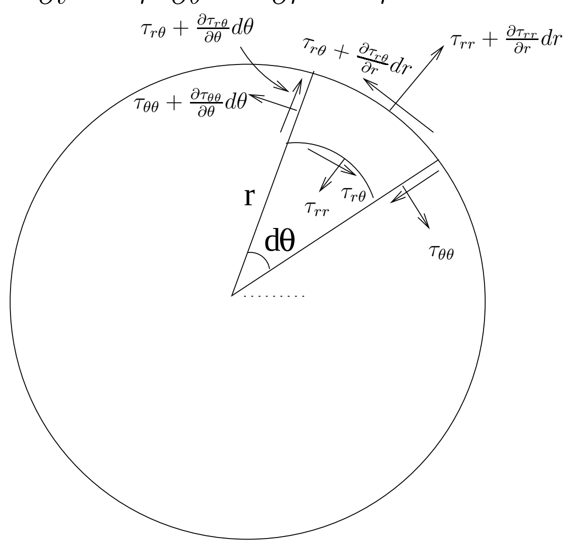

Figure 1: