ME242: Assignment 5

Due: 10/11/14

1. Assuming the Prandtl stress function for the torsion of a bar of equilateral cross section with sides $x + \frac{a}{3} = 0$, $x - \frac{2}{3}a + \sqrt{3}y = 0$, and $x - \frac{2}{3}a - \sqrt{3}y = 0$ to be of the form

$$\phi = m\left(x + \frac{a}{3}\right)\left(x - \frac{2}{3}a + \sqrt{3}y\right)\left(x - \frac{2}{3}a - \sqrt{3}y\right),$$

find m. Using this expression for m,

   (a) find the value of the stresses at the corners and the centroid of the triangle.

   (b) find the maximum value of the shear stress (magnitudewise), and the position where it occurs.

   (c) find an expression for the torsional rigidity C.

2. For the torsion of a circular bar of radius a with a circular cutout of radius b (see Fig. 1), the conjugate harmonic function is given by

$$g = ar\cos\theta - \frac{b^2 a}{r}\cos\theta + \frac{1}{2}b^2.$$

   Show that $g = \frac{1}{2}(x^2 + y^2) = \frac{r^2}{2}$ is satisfied on the boundary. Find the warping function $\psi(r, \theta)$. (Hint: Find an appropriate complex function $W(z)$ whose imaginary part is given by g). Using these functions, find the maximum and minimum shearing stresses, and the positions where they occur.

3. Find an expression for the torsional rigidity of a hollow elliptical section bar, whose outer and inner boundaries are similar ellipses, i.e., $a_1/a_2 = b_1/b_2 = q$ (see Fig. 2).

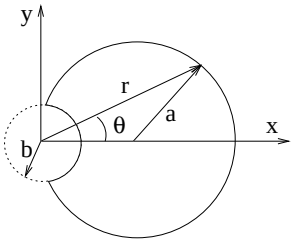

Figure 1:

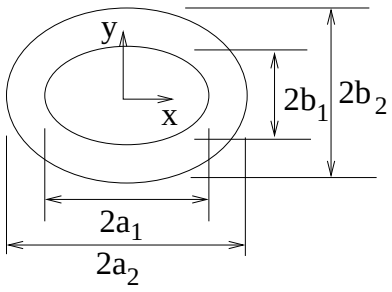

Figure 2: