ME242: Assignment 6

Due: 24/11/14

1. A beam with an equilateral cross section is loaded by a statically equivalent load of P acting at the centroid and directed along the x-axis as shown in Fig. 1. By symmetry, $x_{\text{cf}} = 0$. Show that $y_{\text{cf}} = 0$, i.e., the shear center is at the centroid, by using the following procedure:

   (a) State the value of I_{xy} (without proof), and use it to find the 'curvatures'

   $$\kappa_x = \frac{I_{xx}W_x + I_{xy}W_y}{E(I_{xx}I_{yy} - I_{xy}^2)}; \quad \kappa_y = \frac{I_{yy}W_y + I_{xy}W_x}{E(I_{xx}I_{yy} - I_{xy}^2)},$$

   in terms of I_{xx} and I_{yy} (which you need not evaluate).

   (b) Find the functions $g(x)$ and $f(y)$ such that the boundary conditions

   $$\left[\frac{1}{2}E\kappa_x x^2 - f(y)\right]\frac{dy}{ds} = 0; \quad \left[\frac{1}{2}E\kappa_y y^2 + g(x)\right]\frac{dx}{ds} = 0,$$

   are satisfied. (Hint: See if $f(y)$ can be determined by considering part of the surface).

   (c) Given that $\nu = 0.5$, use the governing differential equation

   $$\boldsymbol{\nabla}^2\phi = -2G\nu\kappa_y x - \frac{\partial g}{\partial x} + 2G\nu\kappa_x y - \frac{\partial f}{\partial y},$$

   to find ϕ (Hint: $\phi = 0$ on the boundary). Substitute this expression for ϕ in the equation

   $$y_{\text{cf}} = -\int_A \left[\frac{2\phi}{W_x} - \frac{I_{xx}}{I_{xx}I_{yy} - I_{xy}^2}x^2 y + \frac{I_{xy}}{I_{xx}I_{yy} - I_{xy}^2}xy^2\right] dA,$$

   to find the location of the shear center. If the load P passes through the centroid, then the same ϕ that is determined above can be used to find the stresses.

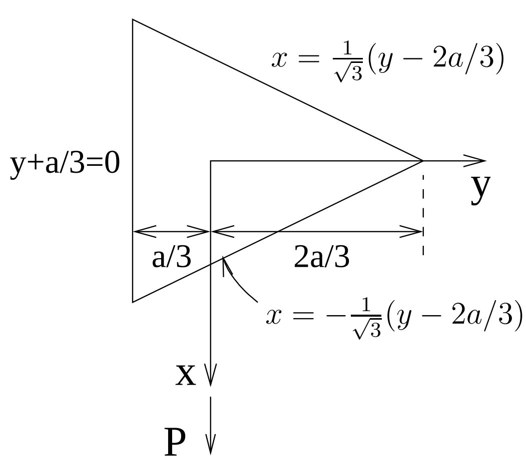

Figure 1: Problem 1.

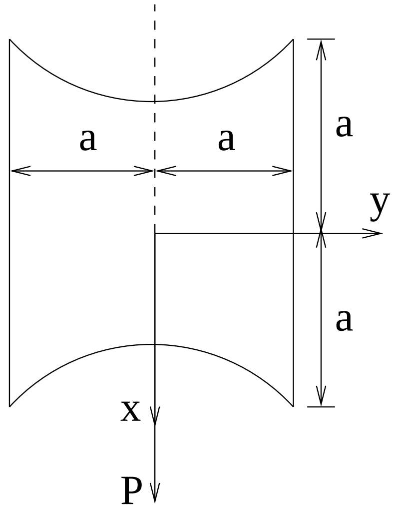

Figure 2: Problem 2.

2. A beam of the doubly symmetric cross section comprised of two vertical sides $y = \pm a$, and two hyperbolas $(1+\nu)x^2 - \nu y^2 = a^2$ is loaded by a statically equivalent load of P acting at the centroid and directed along the x-axis as shown in Fig. 2.

   (a) State with justification (but without mathematical proof), the value of twist at the centroid α.

   (b) State the value of I_{xy} (without proof), and use it to find the 'curvatures'

   $$\kappa_x = \frac{I_{xx}W_x + I_{xy}W_y}{E(I_{xx}I_{yy} - I_{xy}^2)}; \quad \kappa_y = \frac{I_{yy}W_y + I_{xy}W_x}{E(I_{xx}I_{yy} - I_{xy}^2)},$$

   in terms of I_{xx} and I_{yy} (which you need not evaluate).

   (c) Find the functions $g(x)$ and $f(y)$ such that the boundary conditions

   $$\left[\frac{1}{2}E\kappa_x x^2 - f(y)\right]\frac{dy}{ds} = 0; \quad \left[\frac{1}{2}E\kappa_y y^2 + g(x)\right]\frac{dx}{ds} = 0,$$

   are satisfied.

   (d) Use the governing differential equation

   $$\boldsymbol{\nabla}^2\phi = -2G\nu\kappa_y x - \frac{\partial g}{\partial x} + 2G\nu\kappa_x y - \frac{\partial f}{\partial y},$$

   and the appropriate boundary condition to find ϕ (Hint: Take $\phi = a_1 + a_2 x + a_3 y$, and find a_1, a_2 and a_3.)

   (e) Find the stresses using the relations

   $$\tau_{xz} = \frac{\partial\phi}{\partial y} + f(y) - \frac{1}{2}E\kappa_x x^2,$$
   $$\tau_{yz} = -\frac{\partial\phi}{\partial x} - g(x) - \frac{1}{2}E\kappa_y y^2,$$

   and find $(\tau_{xz})_{\max}$, and the location where it occurs.