Indian Institute of Science

ME 242: Final Exam

Date: 5/12/11.
Duration: 9.30 a.m.–12.30 p.m.
Maximum Marks: 100

1. A circular cylinder of radius a and length L is subjected to a prescribed displacement $u_z = \bar{u}$ at its ends as shown in Fig. 1. Assume that the circular cylinder remains circular and prismatic after deformation. Make an intelligent guess of the form of (u_r, u_θ, u_z) as a function of (r, θ, z), and use that to find the displacement and stress fields. (Hint: Are the stresses and strains constant?) (20)

2. A truncated wedge of unit thickness (in the z-direction) is loaded by a traction distribution on the top and bottom edges, whose resultant is an axial total load P, as shown in Fig. 2. The lateral edges $\theta = \theta_0$ are traction-free. The stress distribution is given by (30)

$$
\begin{aligned}
\tau_{rr} &= \frac{c_1 \cos\theta}{r}, \\
\tau_{\theta\theta} &= c_2 \log r + \frac{c_3 \sin\theta}{r}, \\
\tau_{r\theta} &= c_4 r,
\end{aligned}
$$

with the remaining stresses zero (plane-stress conditions). Treating this problem as two-dimensional, and using *only* the boundary conditions, find the constants c_1, c_2, c_3 and c_4.

3. A truncated hollow sphere with inner and outer radii R_1 and R_2, is subjected to torsional loading as shown in Fig. 3. The torque, which is generated by a traction distribution on the flat ends, is given by $T\boldsymbol{e}_z$, and the stress (25)

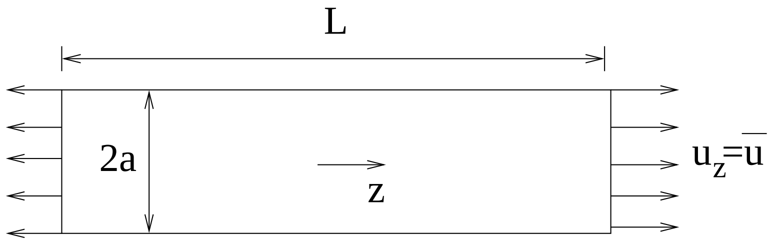

Figure 1: Circular cylinder subjected to a prescribed end displacement $u_z = \bar{u}$

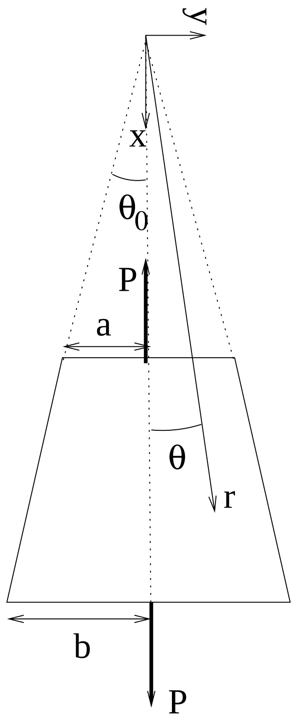

Figure 2: Truncated wedge loaded by an axial load P

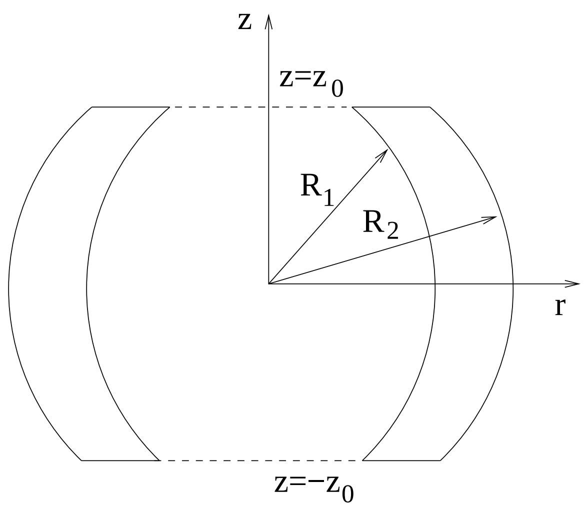

Figure 3: Truncated hollow sphere subjected to torsion.

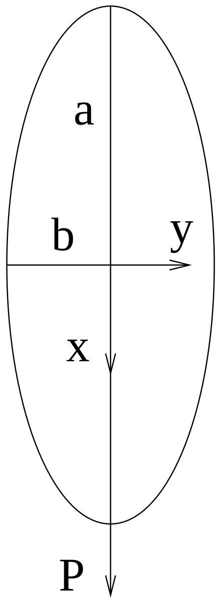

Figure 4: Beam loaded by a terminal load.

distribution is given by

$$\tau_{r\theta} = -\frac{3c}{r^2} z\sqrt{r^2+z^2},$$
$$\tau_{\theta z} = \frac{3c}{r}\sqrt{r^2+z^2},$$

with the other stresses zero. Find c in terms of T, z_0, R_1 and R_2.

4. A beam of length L whose cross-section is given by the equation (25)

$$\left(1-\frac{x^2}{a^2}\right)^{2\nu} = \left(\frac{y}{b}\right)^2,$$

where ν is the Poisson ratio, is loaded by a traction distribution on the end-face $z = L$ with resultant $P\boldsymbol{e}_x$, as shown in Fig. 4. The lateral surfaces are traction-free. Assuming

$$\tau_{xz} = c_1(a^2 - x^2),$$
$$\tau_{yz} = c_2 xy,$$
$$\tau_{zz} = -E(L-z)(\kappa_x x + \kappa_y y),$$

where

$$\kappa_x = \frac{I_{xx}W_x + I_{xy}W_y}{E(I_{xx}I_{yy} - I_{xy}^2)}, \quad \kappa_y = \frac{I_{xy}W_x + I_{yy}W_y}{E(I_{xx}I_{yy} - I_{xy}^2)},$$

and after making appropriate assumptions on the nature of the other stress components, determine c_1 and c_2 using the equations of equilibrium, and the boundary conditions (do not find I_{xx} or I_{yy} explicitly in terms of a and b). State (without proof, but with justification), the values of $\int_A \tau_{xz}\, dA$, $\int_A \tau_{yz}\, dA$ and $\int_A (x\tau_{yz} - y\tau_{xz})\, dA$.

Some relevant formulae

$$\epsilon_{rr} = \frac{\partial u_r}{\partial r}, \qquad \epsilon_{r\theta} = \frac{1}{2}\left[\frac{1}{r}\frac{\partial u_r}{\partial \theta} + r\frac{\partial}{\partial r}\left(\frac{u_\theta}{r}\right)\right],$$

$$\epsilon_{\theta\theta} = \frac{1}{r}\left(\frac{\partial u_\theta}{\partial \theta} + u_r\right), \qquad \epsilon_{\theta z} = \frac{1}{2}\left(\frac{\partial u_\theta}{\partial z} + \frac{1}{r}\frac{\partial u_z}{\partial \theta}\right),$$

$$\epsilon_{zz} = \frac{\partial u_z}{\partial z}, \qquad \epsilon_{rz} = \frac{1}{2}\left(\frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r}\right).$$

$$\boldsymbol{\tau} = \lambda(\operatorname{tr}\boldsymbol{\epsilon})\boldsymbol{I} + 2\mu\boldsymbol{\epsilon}.$$

$$\frac{\partial \tau_{rr}}{\partial r} + \frac{1}{r}\frac{\partial \tau_{r\theta}}{\partial \theta} + \frac{\partial \tau_{rz}}{\partial z} + \frac{\tau_{rr} - \tau_{\theta\theta}}{r} = 0,$$

$$\frac{\partial \tau_{\theta r}}{\partial r} + \frac{1}{r}\frac{\partial \tau_{\theta\theta}}{\partial \theta} + \frac{\partial \tau_{\theta z}}{\partial z} + \frac{\tau_{r\theta} + \tau_{\theta r}}{r} = 0,$$

$$\frac{\partial \tau_{zr}}{\partial r} + \frac{1}{r}\frac{\partial \tau_{z\theta}}{\partial \theta} + \frac{\partial \tau_{zz}}{\partial z} + \frac{\tau_{zr}}{r} = 0.$$

$$[\bar{\boldsymbol{v}}] = \boldsymbol{Q}[\boldsymbol{v}]; \quad [\bar{\boldsymbol{\tau}}] = \boldsymbol{Q}[\boldsymbol{\tau}]\boldsymbol{Q}^T, \text{ where } Q_{ij} = \bar{\boldsymbol{e}}_i \cdot \boldsymbol{e}_j$$

$$\int_{-a}^{a} \frac{dy}{(a^2\cot^2\theta_0 + y^2)} = \frac{2\theta_0\tan\theta_0}{a},$$

$$\int_{-a}^{a} \frac{dy}{(a^2\cot^2\theta_0 + y^2)^2} = \frac{\tan^3\theta_0}{a^3}\left[\theta_0 + \sin\theta_0\cos\theta_0\right].$$