Indian Institute of Science

ME 242: Final Exam

Date: 3/12/04.
Duration: 9.30 a.m.–12.30 p.m.
Maximum Marks: 100

1. A hollow disc is spinning with an angular velocity $\omega = 10$ rad/sec, as shown in Fig. 1. The outer boundary is constrained against radial displacement. (20)

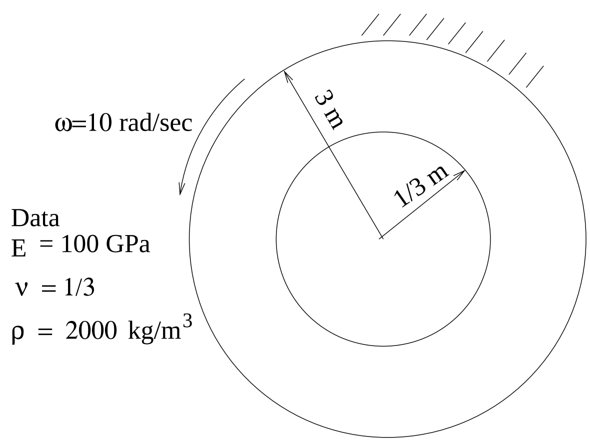

Figure 1: Spinning hollow disc.

   (a) Derive the governing equation for displacement in rotating disc problems. Apply this formulation to a rotating hollow disc free at the inner boundary, and fixed at the outer boundary.
   (b) Calculate the stresses σ_{rr} and $\sigma_{\theta\theta}$ at $r = 1$ m.

2. The curl of a second-order tensor $\boldsymbol{T}$, denoted by $\boldsymbol{\nabla} \times \boldsymbol{T}$, is given by (20)

$$(\boldsymbol{\nabla} \times \boldsymbol{T})_{ij} = \epsilon_{irs} \frac{\partial T_{js}}{\partial x_r}.$$

   (a) If $\boldsymbol{S} \in \mathrm{Sym}$, find an expression for the components of $\boldsymbol{\nabla} \times (\boldsymbol{\nabla} \times \boldsymbol{S})$.
   (b) Use this expression to find if (i) $\boldsymbol{\nabla} \times (\boldsymbol{\nabla} \times \boldsymbol{S})$ is symmetric, and (ii) $\boldsymbol{\nabla} \cdot [\boldsymbol{\nabla} \times (\boldsymbol{\nabla} \times \boldsymbol{S})]$ is zero.
   (c) Let $\boldsymbol{a}$ be a vector such that $\boldsymbol{\nabla}^2 \boldsymbol{a} = -\rho \boldsymbol{b}$, where $\boldsymbol{b}$ is the body force per unit mass, and let $\boldsymbol{S}$ be an arbitrary symmetric tensor. Verify whether the stress tensor $\boldsymbol{\tau}$ given by

$$\boldsymbol{\tau} = \boldsymbol{\nabla} \times (\boldsymbol{\nabla} \times \boldsymbol{S}) + (\boldsymbol{\nabla} \boldsymbol{a}) + (\boldsymbol{\nabla} \boldsymbol{a})^t - (\boldsymbol{\nabla} \cdot \boldsymbol{a}) \boldsymbol{I},$$

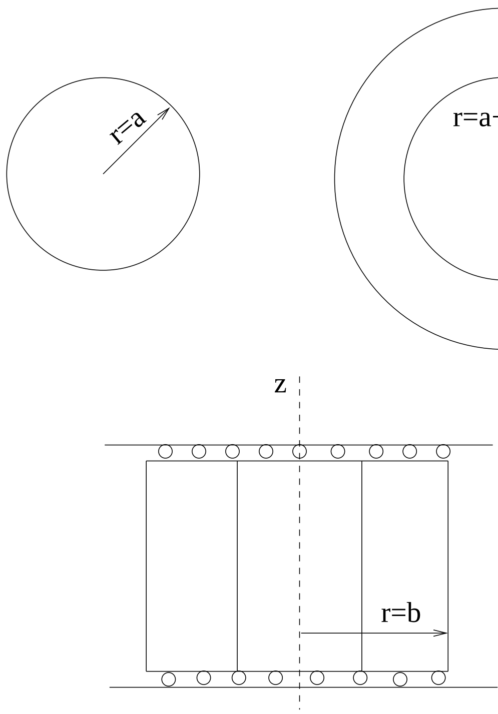

Figure 2: Problem 3.

satisfies the equations of equilibrium and symmetry, i.e., whether it satisfies

$$\boldsymbol{\nabla} \cdot \boldsymbol{\tau} + \rho \boldsymbol{b} = \boldsymbol{0},$$
$$\boldsymbol{\tau} = \boldsymbol{\tau}^t.$$

3. A hollow cylinder of inner radius $a - \Delta$ and outer radius b is shrink fitted around a solid cylinder of radius a as shown in Fig. 2 (the individual cylinders *before* shrinkfitting are shown in the upper part of the figure, and the assembly *after* shrinkfitting is shown in the lower part). Both cylinders are made of the same material. Plane strain conditions are maintained during the shrinkfitting process by constraining the motion along the z-direction (but *not* along the other directions) as shown in the figure, i.e., $u_z = \epsilon_{zz} = 0$. Assuming the process to be axiymmetric, i.e., $u_r = u_r(r)$, the Navier equations reduce to (30)

$$\frac{d}{dr}\left[\frac{1}{r}\frac{d(ru_r)}{dr}\right] = 0.$$

   (a) Find a general solution to the above equation.

   (b) Write the appropriate displacement and stress boundary conditions. Assuming Δ to be small in comparsion to a and b, you may state the boundary conditions at $r = a$ and $r = b$. (Hint: Be careful of signs for boundary condtions on u_r)

   (c) Use the strain-displacement relations $\epsilon_{rr} = \partial u_r/\partial r$ and $\epsilon_{\theta\theta} = u_r/r$ (you may assume the other strain components to be zero), and the constitu-

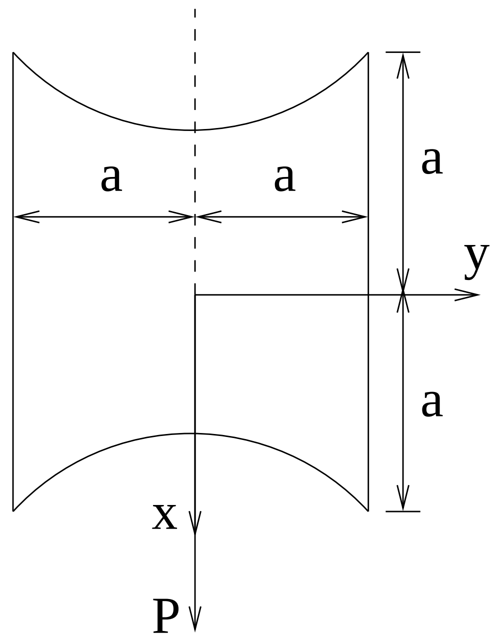

Figure 3: Problem 4.

tive relation $\boldsymbol{\tau} = \lambda(\mathrm{tr}\,\boldsymbol{\epsilon})\boldsymbol{I} + 2\mu\boldsymbol{\epsilon}$ to find expressions for the displacement and stress fields in the solid and hollow cylinders as a function of Δ (you *need not* convert (λ, μ) to (E, ν)).

(d) Find the *total* axial force F_z exerted by the top wall on the top surface of the total cylinder.

4. A beam of the doubly symmetric cross section comprised of two vertical sides $y = \pm a$, and two hyperbolas $(1+\nu)x^2 - \nu y^2 = a^2$ is loaded by a statically equivalent load of P acting at the centroid and directed along the x-axis as shown in Fig. 3. (30)

(a) State with justification (but without mathematical proof), the value of twist at the centroid α.

(b) State the value of I_{xy} (without proof), and use it to find the 'curvatures'

$$\kappa_x = \frac{I_{xx}W_x + I_{xy}W_y}{E(I_{xx}I_{yy} - I_{xy}^2)}; \quad \kappa_y = \frac{I_{yy}W_y + I_{xy}W_x}{E(I_{xx}I_{yy} - I_{xy}^2)},$$

in terms of I_{xx} and I_{yy} (which you need not evaluate).

(c) Find the functions $g(x)$ and $f(y)$ such that the boundary condition

$$\left[\frac{1}{2}E\kappa_x x^2 - f(y)\right]\frac{dy}{ds} - \left[\frac{1}{2}E\kappa_y y^2 + g(x)\right]\frac{dx}{ds} = 0$$

is satisfied.

(d) Use the governing differential equation

$$\nabla^2\phi = -2G\nu\kappa_y x - \frac{\partial g}{\partial x} + 2G\nu\kappa_x y - \frac{\partial f}{\partial y},$$

and the appropriate boundary condition to find ϕ (Hint: Take $\phi = a_1 + a_2 x + a_3 y$, and find a_1, a_2 and a_3.)

(e) Find the stresses using the relations

$$\tau_{xz} = \frac{\partial \phi}{\partial y} + f(y) - \frac{1}{2}E\kappa_x x^2,$$
$$\tau_{yz} = -\frac{\partial \phi}{\partial x} - g(x) - \frac{1}{2}E\kappa_y y^2,$$

and find $(\tau_{xz})_{\max}$, and the location where it occurs.