Indian Institute of Science

ME 242: Final Exam

Date: 5/12/05.
Duration: 9.00 a.m.–12.00 noon.
Maximum Marks: 100

1. A *very thin* rod is fixed inside a spinning hollow disc (a=1 m; b=1.1 m, E=200 GPa, $\nu = 1/3$) as shown in Fig. 1. If the strength of the rod is 100 MPa, calculate the maximum operating speed N in rpm. (20)

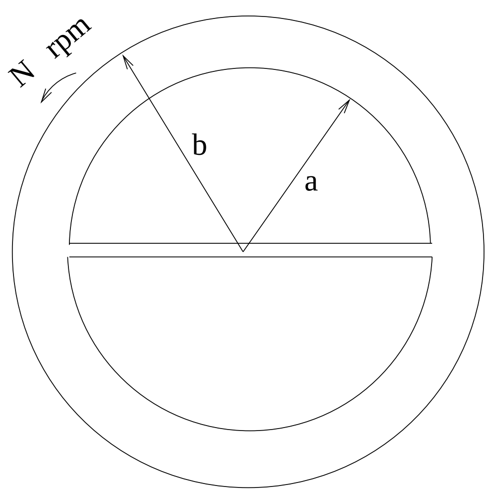

Figure 1: Spinning hollow disc.

2. A second-order symmetric tensor $\boldsymbol{S}$ is said to be positive definite if $(\boldsymbol{u}, \boldsymbol{S}\boldsymbol{u}) \geq 0 \ \forall \boldsymbol{u}$, with equality achieved if and only if $\boldsymbol{u} = \boldsymbol{0}$. Show that $\boldsymbol{S}$ is positive definite if and only if each of its eigenvalues is strictly positive. (10)

3. A body in equilibrium occupies a volume V with surface S. (40)

   (a) By integrating the equation

$$(\boldsymbol{\nabla}_x \cdot \boldsymbol{\tau}) \otimes \boldsymbol{x} + \rho \boldsymbol{b} \otimes \boldsymbol{x} = \boldsymbol{0},$$

   over the domain V, and with the use of the divergence theorem, find a relation between $\int_V \boldsymbol{\tau}\, dV$, $\int_S \boldsymbol{t} \otimes \boldsymbol{x}\, dS$ and $\int_V \rho \boldsymbol{b} \otimes \boldsymbol{x}\, dV$. (Hint: Use indicial notation.)

   (b) For a prismatic bar with uniform density ρ standing vertically on a smooth horizontal surface, which coincides with the xy plane as shown in Fig. 2, and with $\boldsymbol{b} = -g\boldsymbol{e}_z$, find the average values of τ_{zx} and τ_{zz}

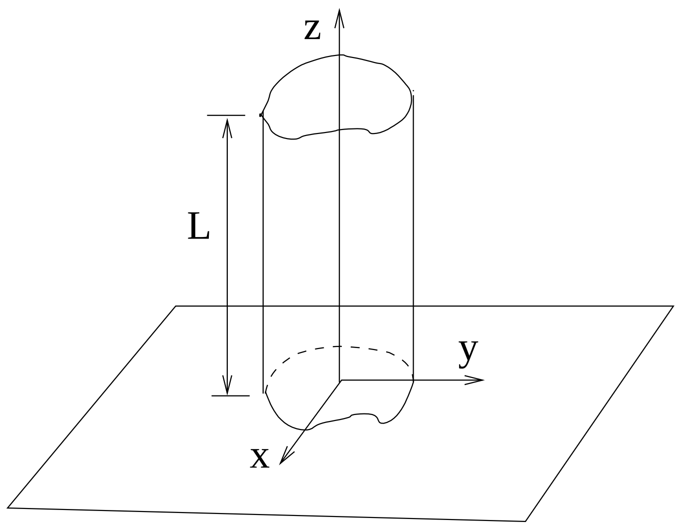

Figure 2: Prismatic bar standing on a smooth horizontal surface.

using the relation you have derived in part (a), where the average value of τ_{ij} is defined as $\int_V \tau_{ij}\, dV/V$. Note that the lateral and top surfaces of the bar are traction free, and that since the contact surface is smooth, there are no horizontal tractions exerted on the bottom of the prismatic bar.

4. Find the center of flexure component $y_{\rm cf}$ (by symmetry, $x_{\rm cf} = 0$), for a bar of semicircular cross section shown in Fig. 3, and for which $\nu = 0$, as outlined below. The equation of the semicircle is given by $x^2 + (y + \frac{4a}{3\pi})^2 = a^2$. Although the formulation is carried out in terms of x and y, *evaluate all the integrals that arise by using the transformation* $x = r\cos\theta$ *and* $y + \frac{4a}{3\pi} = r\sin\theta$, where r and θ are as shown in the figure. (30)

   (a) Show that $I_{xy} = -\int_A xy\, dA = 0$, and $I_{yy} = \int_A x^2\, dA = \pi a^4/8$.

   (b) Since only $y_{\rm cf}$ is to be evaluated, set the appropriate load component from among (W_x, W_y) to zero, and find the 'curvatures'

$$\kappa_x = \frac{I_{xx}W_x + I_{xy}W_y}{E(I_{xx}I_{yy} - I_{xy}^2)}; \quad \kappa_y = \frac{I_{yy}W_y + I_{xy}W_x}{E(I_{xx}I_{yy} - I_{xy}^2)}.$$

   (c) Find the functions $g(x)$ and $f(y)$ such that the boundary condition

$$\left[\frac{1}{2}E\kappa_x x^2 - f(y)\right]\frac{dy}{ds} - \left[\frac{1}{2}E\kappa_y y^2 + g(x)\right]\frac{dx}{ds} = 0$$

   is satisfied.

   (d) Recall that $\nu = 0$, make an appropriate choice of α, and use the governing differential equation

$$\nabla^2\phi = -2G\nu\kappa_y x - \frac{dg}{dx} + 2G\nu\kappa_x y - \frac{df}{dy} - 2G\alpha,$$

   and the appropriate boundary condition to find ϕ.

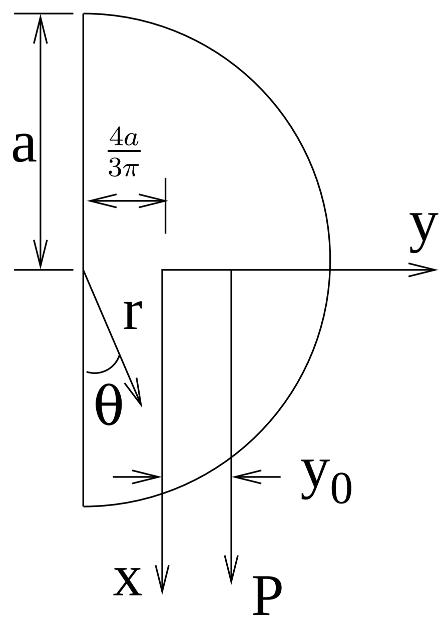

Figure 3: Problem 4.

(e) Use

$$y_{\mathrm{cf}} = -\int_A \left[\frac{2\phi}{W_x} - \frac{I_{xx}}{I_{xx}I_{yy} - I_{xy}^2} x^2 y + \frac{I_{xy}}{I_{xx}I_{yy} - I_{xy}^2} x y^2 \right] dA,$$

and show that $y_{\mathrm{cf}} = ha/\pi$, where h is a constant you should determine.

Some relevant formulae

$$\int_0^\pi \cos\theta \, d\theta = 0,$$

$$\int_0^\pi \sin\theta \, d\theta = 2,$$

$$\int_0^\pi \cos\theta \sin\theta \, d\theta = 0,$$

$$\int_0^\pi \cos^2\theta \, d\theta = \frac{\pi}{2},$$

$$\int_0^\pi \cos^2\theta \sin\theta \, d\theta = \frac{2}{3}.$$