Indian Institute of Science

ME 242: Final Exam

Date: 5/12/06.
Duration: 9.30 a.m.–12.30 p.m.
Maximum Marks: 100

1. The stress at the **center** of a spinning compound disc shown in Fig. 1 is 100 MPa. (20)

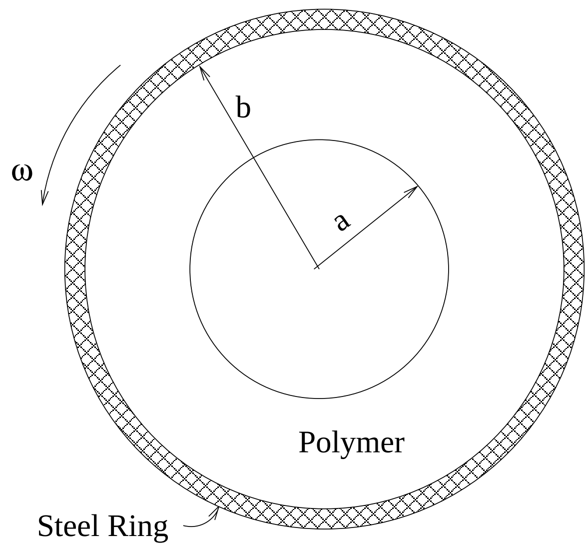

Figure 1: Spinning compound disc.

Given the displacement solution

$$u = c_1 r + c_2/r - (1-\nu^2)\rho\omega^2 r^3/8E,$$

calculate the stresses at the interface ($r = 1$ m) and at the rim ($r = \sqrt{2}$ m). Tabulate your results as follows:

r (m)	Material	σ_r (MPa)	σ_θ (MPa)
0	A	100	?
1	A	?	?
1	B	?	?
$\sqrt{2}$	B	?	?

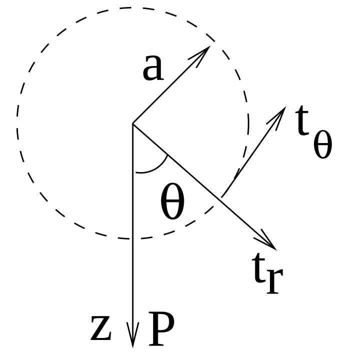

Figure 2: Point load acting in an infinite elastic body.

2. The Navier equations of elasticity for elastostatics in the absence of body forces can be written as (30)

$$\boldsymbol{\nabla}(\boldsymbol{\nabla}\cdot\boldsymbol{u}) + (1-2\nu)\boldsymbol{\nabla}^2\boldsymbol{u} = \boldsymbol{0}, \tag{1}$$

where ν is the Poisson ratio. Let α and $\boldsymbol{\beta}$ be scalar and vector functions (of position) that satisfy

$$\begin{aligned} \boldsymbol{\nabla}^2\alpha &= 0, \\ \boldsymbol{\nabla}^2\boldsymbol{\beta} &= \boldsymbol{\nabla}\cdot(\boldsymbol{\nabla}\boldsymbol{\beta}) = \boldsymbol{0}. \end{aligned}$$

Let $\boldsymbol{u}$ be given in terms of α and $\boldsymbol{\beta}$ as

$$\boldsymbol{u} = \boldsymbol{\nabla}\alpha + (\boldsymbol{\nabla}\boldsymbol{\beta})^T\boldsymbol{x} + m\boldsymbol{\beta}. \tag{2}$$

Find the value of the constant m (in terms of ν) such that the representation given by Eqn. (2) automatically satisfies Eqn. (1).

3. A point load P acts at the origin in the z-direction in an infinite elastic body (see Fig. 2, where the tractions t_r and t_θ are shown). The displacements and nonzero stresses with respect to a *spherical* coordinate system are given by (30)

$$\begin{aligned} u_r &= \frac{2A(1-\nu)\cos\theta}{\mu r}, \\ u_\theta &= -\frac{A(3-4\nu)\sin\theta}{2\mu r}, \\ \tau_{rr} &= -\frac{A(4-2\nu)\cos\theta}{r^2}, \\ \tau_{\theta\theta} &= \frac{A(1-2\nu)\cos\theta}{r^2}, \\ \tau_{r\theta} &= \tau_{\phi\phi} = \frac{A(1-2\nu)\sin\theta}{r^2}. \end{aligned}$$

Considering the vertical equilibrium of the imaginary sphere of radius a shown in Fig. 2, determine the constant A.

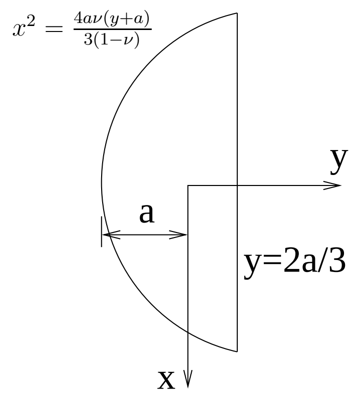

Figure 3: Bar of semi-parabolic cross section.

4. Find the center of flexure component y_{cf} (by symmetry, $x_{\text{cf}} = 0$), for a bar of parabolic cross section shown in Fig. 3. (20)

(a) Show that $I_{yy} = \int_A x^2\,dA = m\left[\frac{\nu}{(1-\nu)}\right]^{3/2} a^4$, where m is a constant you have to determine.

(b) Since only y_{cf} is to be evaluated, set the appropriate load component from among (W_x, W_y) to zero, and find the 'curvatures'

$$\kappa_x = \frac{I_{xx}W_x + I_{xy}W_y}{E(I_{xx}I_{yy} - I_{xy}^2)}; \quad \kappa_y = \frac{I_{yy}W_y + I_{xy}W_x}{E(I_{xx}I_{yy} - I_{xy}^2)}.$$

(c) Find the functions $g(x)$ and $f(y)$ such that the boundary conditions

$$\left[\frac{1}{2}E\kappa_x x^2 - f(y)\right]\frac{dy}{ds} = 0, \quad \left[\frac{1}{2}E\kappa_y y^2 + g(x)\right]\frac{dx}{ds} = 0,$$

are satisfied.

(d) Make an appropriate choice of α, and use the governing differential equation

$$\boldsymbol{\nabla}^2\phi = -2G\nu\kappa_y x - \frac{dg}{dx} + 2G\nu\kappa_x y - \frac{df}{dy} - 2G\alpha,$$

and the appropriate boundary condition to find ϕ.

(e) Using

$$y_{\text{cf}} = -\int_A \left[\frac{2\phi}{W_x} - \frac{I_{xx}}{I_{xx}I_{yy} - I_{xy}^2}x^2 y + \frac{I_{xy}}{I_{xx}I_{yy} - I_{xy}^2}xy^2\right] dA,$$

set up the integral (with the proper limits) to evaluate y_{cf}. *Do not evaluate the integral.*

Some relevant formulae

The rotation matrix which takes a Cartesian system to a spherical system, i.e., $[\boldsymbol{u}]_{\text{spherical}} = [\boldsymbol{Q}][\boldsymbol{u}]_{\text{Cartesian}}$, etc. is given by

$$\boldsymbol{Q} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix},$$

The surface area of a sphere of radius a is

$$\int_0^\pi (2\pi a^2 \sin\theta)\, d\theta = 4\pi a^2.$$

$$\int_0^\pi \cos\theta\, d\theta = 0,$$
$$\int_0^\pi \sin\theta\, d\theta = 2,$$
$$\int_0^\pi \cos\theta\sin\theta\, d\theta = 0,$$
$$\int_0^\pi \cos^2\theta\, d\theta = \frac{\pi}{2},$$
$$\int_0^\pi \cos^2\theta\sin\theta\, d\theta = \frac{2}{3},$$
$$\int_0^\pi \sin^3\theta\, d\theta = \frac{4}{3}.$$