Indian Institute of Science

ME 242: Final Exam

Date: 5/12/07.
Duration: 9.30 a.m.–12.30 p.m.
Maximum Marks: 100

1. A thin steel ring is assembled on a hollow polymer disk at 250 K (see Fig. 1). (20)
The operating temperature is 300 K. The radii are related as $b^2 = 3a^2$.

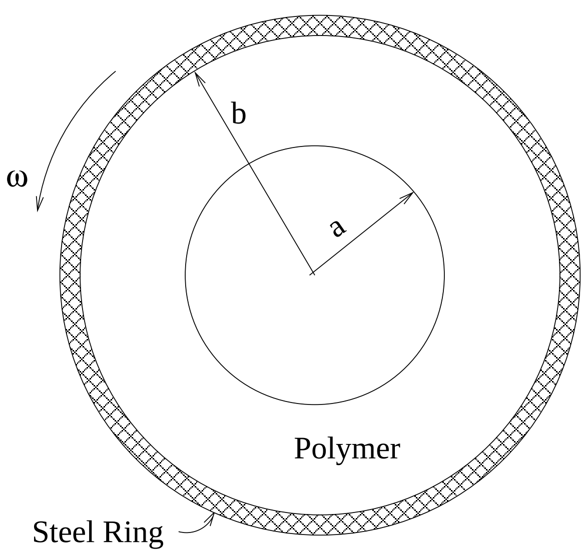

Figure 1: Compound spinning disc.

The properties for the steel disk are $E_s = 200$ GPa, $\rho_s = 8000$ kg/m^3, $\alpha_s = 10^{-5}$/K, while those for the polymer disk are $\nu_p = 1/2$, $E_p = 10$ GPa, $\rho_p = 800$ kg/m^3, $\alpha_p = 85 \times 10^{-6}$/K. Calculate ω at which the ring loses contact with the disc, taking $b =$ .... m.

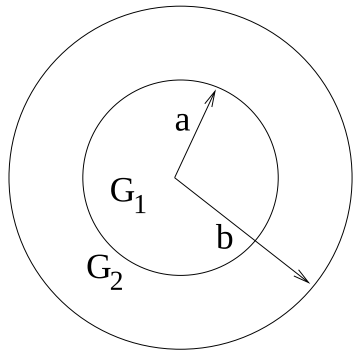

Figure 2: Torsion of a composite bar.

2. The Navier equations of elasticity for an isotropic material , with λ, μ and ρ constant, can be written as (30)

$$(\lambda + \mu)\boldsymbol{\nabla}(\boldsymbol{\nabla} \cdot \boldsymbol{u}) + \mu \boldsymbol{\nabla}^2 \boldsymbol{u} + \rho \boldsymbol{b} = \rho \frac{\partial^2 \boldsymbol{u}}{\partial t^2}. \tag{1}$$

Let $c_L = \sqrt{(\lambda + 2\mu)/\rho}$ and $c_T = \sqrt{\mu/\rho}$ denote the longitudinal and transverse wave speeds.

(a) By taking the divergence of Eqn. (1), determine the constants k_1, k_2, k_3 and k_4 in the relation

$$\left[\boldsymbol{\nabla}^2 - \frac{k_1}{c_L^2}\frac{\partial^2}{\partial t^2} - \frac{k_2}{c_T^2}\frac{\partial^2}{\partial t^2}\right](\boldsymbol{\nabla} \cdot \boldsymbol{u}) + \left(\frac{k_3}{c_L^2} + \frac{k_4}{c_T^2}\right)\boldsymbol{\nabla} \cdot \boldsymbol{b} = 0.$$

(b) By taking the curl of Eqn. (1), determine the constants c_1, c_2, c_3 and c_4 in the relation

$$\left[\boldsymbol{\nabla}^2 - \frac{c_1}{c_L^2}\frac{\partial^2}{\partial t^2} - \frac{c_2}{c_T^2}\frac{\partial^2}{\partial t^2}\right](\boldsymbol{\nabla} \times \boldsymbol{u}) + \left(\frac{c_3}{c_L^2} + \frac{c_4}{c_T^2}\right)\boldsymbol{\nabla} \times \boldsymbol{b} = \boldsymbol{0}.$$

3. We have seen in the class that $u_\theta = \alpha r z$, $\gamma_{\theta z} = \alpha r$, $\tau_{\theta z} = G\gamma_{\theta z} = G\alpha r$, other components zero, is a solution for the torsion of a circular cylinder fixed at $z = 0$. (20)

(a) Evaluate by finding the stress field (which you can directly write by using the solution given above) if the same displacement solution as above is a solution to the problem of torsion of a composite cylinder shown in Fig. 2. The composite cylinder is fixed at $z = 0$, the total torque T is applied (as usual) via tractions on the face $z = L$, and the outermost lateral surface is traction free. (You need not check the equations of equilibrium, but you should check other relevant things such as the total force on the face $z = L$, boundary conditions and so on).

(b) Derive a relation between α and the total torque T in terms of G_1, G_2, and the polar moments of inertia of the inner and outer cylinders, J_1 and J_2.

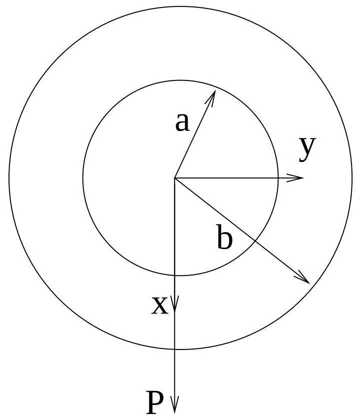

Figure 3: Bending of a hollow circular cylinder by a terminal load P.

4. Consider the problem of flexure by a terminal load P of a hollow circular cylinder with inner and outer radii a and b as shown in Fig. 3. The inner and outer surfaces of the cylinder are traction free. Assuming the Poisson ratio to be zero, the stress distribution, with $I_{yy} = \pi(b^4 - a^4)/4$, is given by (30)

$$\tau_{xz} = c_1 \left[a^2 + b^2 - x^2 + \frac{a^2 b^2}{x^2 + y^2} - \frac{2a^2 b^2 x^2}{(x^2 + y^2)^2} \right] + c_2 y^2,$$
$$\tau_{yz} = -\frac{Pxy}{4I_{yy}} \left[1 + \frac{3a^2 b^2}{(x^2 + y^2)^2} \right],$$
$$\tau_{zz} = (z - L)\frac{Px}{I_{yy}},$$

with other stress components zero.

(a) Using the equations of equilibrium and boundary conditions, find the constants c_1 and c_2. (Hint: If an expression is valid for all x and y, evaluate it at an intelligent choice of x and y.)

(b) Using the solution you have found above, find the total force on the end face in the x and y directions, and verify that they match with the given data.

Some relevant formulae

$$\sin^2\theta = \frac{1}{2}(1 - \cos 2\theta),$$
$$\cos^2\theta = \frac{1}{2}(1 + \cos 2\theta),$$
$$\sin\theta\cos\theta = \frac{1}{2}\sin 2\theta.$$