Indian Institute of Science

ME 242: Final Exam

Date: 3/12/08.
Duration: 9.00 a.m.–12.00 noon
Maximum Marks: 100

1. A thin compound disk of radius $b = \sqrt{2}$ m (see Fig. 1) is spinning with an angular velocity of $\omega = 100$ rad/sec. The inner core material and the outer ring material have the same Young modulus $E_A = E_B = 200$ GPa and Poisson ratio $\nu_A = \nu_B = 0.25$. However, the density of the outer ring material ρ_B is different from the inner core density $\rho_A = 8000$ kg/m^3. Calculate the (20)

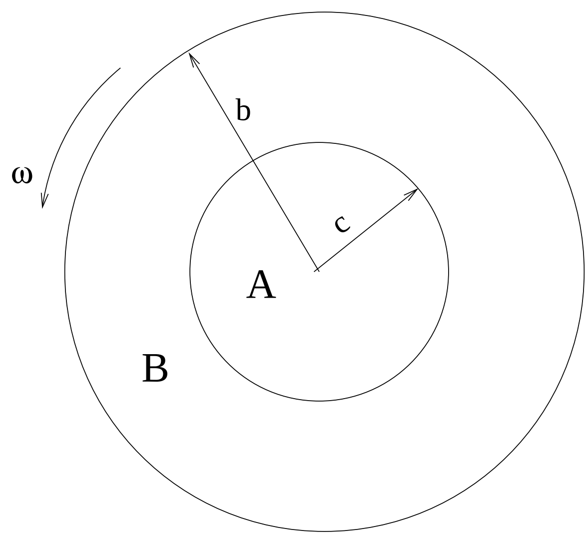

Figure 1: Compound spinning disc.

radial and tangential stresses at the interface with radius $c = 1$ m. The density $\rho_B =$ .... kg/m^3.

2. If $(\gamma_1, \boldsymbol{n}_1)$, $(\gamma_2, \boldsymbol{n}_2)$ and $(\gamma_3, \boldsymbol{n}_3)$ denote the eigenvalue/eigenvectors of $\boldsymbol{\tau}$, find the eigenvalue/eigenvectors of $\boldsymbol{\epsilon}$, where $\boldsymbol{\tau} = \lambda(\mathrm{tr}\,\boldsymbol{\epsilon})\boldsymbol{I} + 2\mu\boldsymbol{\epsilon}$. (25)

3. We saw in the test that the solution to the problem shown in Fig. 2 is given by $\tau_{r\theta} = c/r^2$, where $c = M/(2\pi L)$, with other stress components zero. Using the stress-strain and strain-displacement relations (it is given that the compatibility relations are satisfied), and assuming $u_z = 0$, and (u_r, u_θ) to be independent of z, find the displacement field (u_r, u_θ). (15)

4. A plane stress solution for bending by an end load P of the curved beam (40)

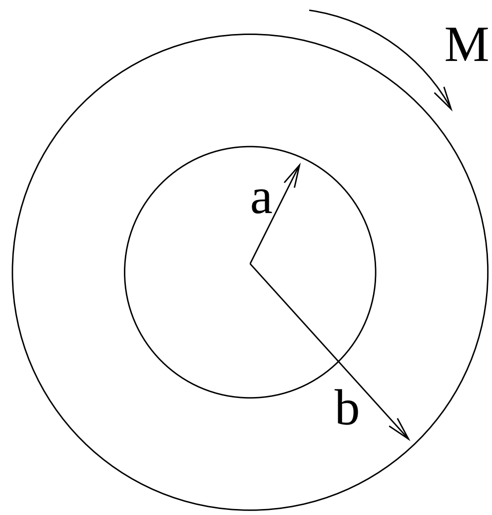

Figure 2: Hollow cylinder of length L fixed at the inner boundary $r = a$, and subjected to a moment M at the outer boundary $r = b$.

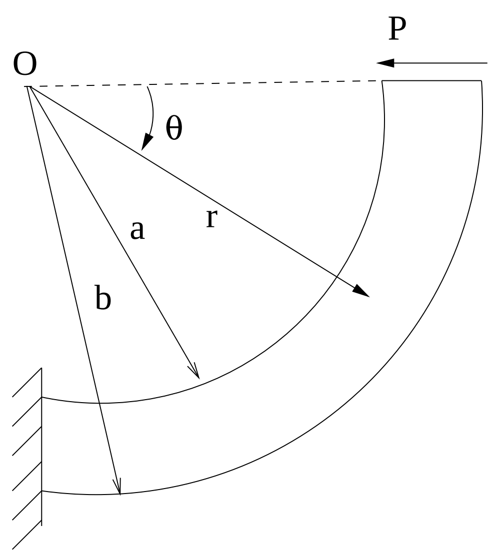

Figure 3: A curved beam subjected to bending by an end load P.

shown in Fig. 3 is given by

$$\tau_{rr} = c_1 \left(r + \frac{k_1}{r^3} + \frac{k_2}{r} \right) \sin\theta,$$
$$\tau_{\theta\theta} = c_2 \left(3r - \frac{k_1}{r^3} + \frac{k_2}{r} \right) \sin\theta,$$
$$\tau_{r\theta} = c_3 \left(r + \frac{k_3}{r^3} + \frac{k_4}{r} \right) \cos\theta.$$

(a) Assuming unit width of cross section, and using the appropriate equation of equilibrium (note that the 'plane stress' assumption is being made, and hence not all equilibrium equations need be considered), and the traction boundary conditions, find the constants c_1, c_2, c_3, k_1, k_2, k_3 and k_4 (all of which are nonzero).

(b) Consider the free-body diagram of the beam from $\theta = 0$ to $\theta = \theta_0$ where $0 < \theta_0 < \pi/2$. Evaluate the tractions at the cut section $\theta = \theta_0$, and find the resultant forces and moment (about the origin O) due to the tractions, at this cut section. Then determine if the piece whose free-body diagram you have drawn is in static equilibrium.

Some relevant formulae

$$\epsilon_{rr} = \frac{\partial u_r}{\partial r}, \qquad \epsilon_{r\theta} = \frac{1}{2}\left[\frac{1}{r}\frac{\partial u_r}{\partial \theta} + r\frac{\partial}{\partial r}\left(\frac{u_\theta}{r}\right) \right],$$
$$\epsilon_{\theta\theta} = \frac{1}{r}\left(\frac{\partial u_\theta}{\partial \theta} + u_r \right), \qquad \epsilon_{\theta z} = \frac{1}{2}\left(\frac{\partial u_\theta}{\partial z} + \frac{1}{r}\frac{\partial u_z}{\partial \theta} \right),$$
$$\epsilon_{zz} = \frac{\partial u_z}{\partial z}, \qquad \epsilon_{rz} = \frac{1}{2}\left(\frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r} \right).$$

$$\boldsymbol{\tau} = \lambda(\mathrm{tr}\,\boldsymbol{\epsilon})\boldsymbol{I} + 2\mu\boldsymbol{\epsilon}.$$
$$\frac{\partial \tau_{rr}}{\partial r} + \frac{1}{r}\frac{\partial \tau_{r\theta}}{\partial \theta} + \frac{\tau_{rr} - \tau_{\theta\theta}}{r} = 0.$$