Indian Institute of Science

ME 242: Midsemester Test

Date: 28/9/02.
Duration: 9.30 a.m.–11.00 a.m.
Maximum Marks: 100

Instructions:

1. State all your assumptions and explanations **clearly**.
2. The points for each question are indicated in the right margin.

1. Using the formulae on pg. 2, show that if $\boldsymbol{x}$ is the position vector of a point, and $\boldsymbol{t}$ is the axial vector of $(\boldsymbol{T} - \boldsymbol{T}^t)$, then (45)

$$\int_V [\boldsymbol{x} \times (\boldsymbol{\nabla} \cdot \boldsymbol{T}) + \boldsymbol{t}] \, dV = \int_S \boldsymbol{x} \times (\boldsymbol{T}\boldsymbol{n}) \, dS.$$

2. This problem leads you through an alternate derivation for the relation between the shear modulus, G, and the Young modulus E and Poisson ratio ν. Let the primed coordinate system, $\{\bar{\boldsymbol{e}}_1, \bar{\boldsymbol{e}}_2, \bar{\boldsymbol{e}}_3\}$, be obtained by a rotation of $\boldsymbol{e}_1$ and $\boldsymbol{e}_2$ by 45^o about the $\boldsymbol{e}_3$ axis as shown in Fig. 1 ($\bar{\boldsymbol{e}}_3 = \boldsymbol{e}_3$). Let the state of stress at a point with respect to the unprimed system be one of uniaxial tension, i.e., (35)

$$[\boldsymbol{\tau}] = \begin{bmatrix} p & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

(a) Using the stress-strain constitutive relations on pg. 2, find the components of the strain tensor with respect to the unprimed coordinate system, i.e., with respect to $\{\boldsymbol{e}_1, \boldsymbol{e}_2, \boldsymbol{e}_3\}$.

(b) Using the second-order tensor transformation relations given at the end, find the components of the stress tensor and the strain tensor in the primed coordinate system.

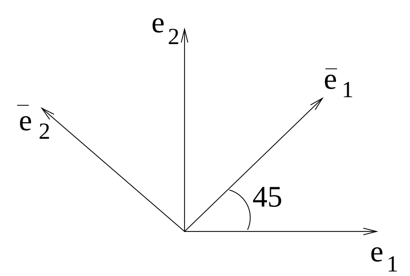

Figure 1:

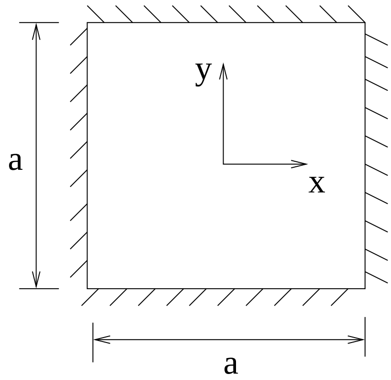

Figure 2:

(c) Now using the constitutive relation in the primed coordinate system, find the required relation between G, E and ν.

3. An elastic material of dimensions $a \times a \times h$ that is restricted along the edges and bottom by smooth rigid walls (see Fig. 2) is subjected to a uniform traction $-p\boldsymbol{e}_z$ on the top surface. Assuming that the only nonzero strain component is ϵ_{zz}, and that it is uniform throughout the body, find ϵ_{zz} and the stresses in the body. Using the stress distribution, find the traction vector on the right edge of the rigid wall, and hence, the total force exerted on it. Neglect body forces. (20)

Some relevant formulae

$$(\boldsymbol{u} \times \boldsymbol{v})_i = \epsilon_{ijk} u_j v_k.$$

If $\boldsymbol{w}$ is the axial vector of a skew-symmetric tensor $\boldsymbol{W}$, then

$$w_i = -\frac{1}{2}\epsilon_{ijk} W_{jk},$$
$$W_{ij} = -\epsilon_{ijk} w_k.$$

Divergence theorem

$$\int_V \frac{\partial \phi}{\partial x_r}\, dV = \int_S \phi n_r \, dS.$$

Stress-Strain relations

$$\epsilon_{xx} = \frac{1}{E}\left[\tau_{xx} - \nu(\tau_{yy} + \tau_{zz})\right],$$
$$\gamma_{xy} = \frac{1}{G}\tau_{xy},$$

with analogous relations for the other components.

Tensor transformation relations

$$[\bar{\boldsymbol{T}}] = \boldsymbol{Q}[\boldsymbol{T}]\boldsymbol{Q}^t$$

where $(\boldsymbol{Q})_{ij} = \bar{\boldsymbol{e}}_i \cdot \boldsymbol{e}_j$.