Indian Institute of Science
ME 242: Midsemester Test

Date: 24/9/11.
Duration: 9.30 a.m.–11.00 a.m.
Maximum Marks: 100

Instructions:

1. Justify all your steps.

2. The points for each question are indicated in the right margin.

1. Let $\boldsymbol{W} \in \text{Skw}$ which has an axial vector $\boldsymbol{w}$ of unit magnitude, i.e., $|\boldsymbol{w}| = \sqrt{\boldsymbol{w}\cdot\boldsymbol{w}} = 1$, and let (30)
$$\boldsymbol{T} = \boldsymbol{I} + \sin\alpha\,\boldsymbol{W} + (1-\cos\alpha)\boldsymbol{W}^2.$$
Using the relation $\boldsymbol{W}\boldsymbol{u} = \boldsymbol{w}\times\boldsymbol{u}$, first find $\boldsymbol{T}\boldsymbol{u}$, and then $|\boldsymbol{T}\boldsymbol{u}| - |\boldsymbol{u}|$ (You may directly use $\boldsymbol{a}\times(\boldsymbol{b}\times\boldsymbol{c}) = (\boldsymbol{a}\cdot\boldsymbol{c})\boldsymbol{b} - (\boldsymbol{a}\cdot\boldsymbol{b})\boldsymbol{c}$. *Derive* any other relations that you need on the way). What can you conclude about the nature of $\boldsymbol{T}$? Guess one eigenvector of $\boldsymbol{T}$.

2. Let the displacement field be $\boldsymbol{u}(\boldsymbol{x},t) = f(|\boldsymbol{x}|)\boldsymbol{x}\cos(\omega t)$, where $\boldsymbol{x}$ is the position vector, and $|\boldsymbol{x}| = \sqrt{\boldsymbol{x}\cdot\boldsymbol{x}}$. By substituting $\boldsymbol{u}$ in the Navier equations of elasticity under zero body forces given by (40)
$$\left(\frac{\partial^2\boldsymbol{u}}{\partial t^2}\right)_{\boldsymbol{x}} = (\lambda+\mu)\boldsymbol{\nabla}(\boldsymbol{\nabla}\cdot\boldsymbol{u}) + \mu\boldsymbol{\nabla}^2\boldsymbol{u},$$
find the governing ordinary differential equation for $f(|\boldsymbol{x}|)$ (*do not* attempt to solve this equation).

3. A circular cylinder of length L and radius b, is subjected to a uniform longitudinal traction T, and a constant pressure $p = p_0$ at the outer surface $r = b$, as shown in Fig. 1. By assuming the body forces to be zero, and the displacement field to be given by (30)
$$2\mu u_r = \left[-\frac{T}{2} - (1-\nu)(c_1 + 2c_2 z)\right] r$$
$$2\mu u_z = (T + 2\nu c_1)z,$$
use the strain-displacement and stress-strain relations, and boundary conditions to find the constants c_1 and c_2 in terms of ν, T and p_0. Note that you need *not* check the equilibrium equations since the given displacement field satisfies them.

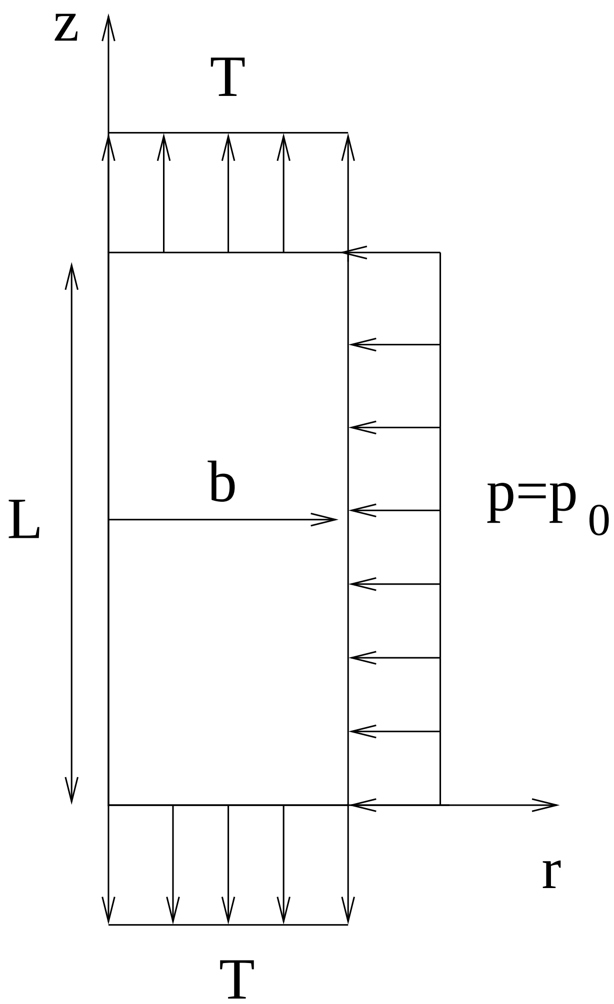

Figure 1: A circular cylinder of radius b and length L subjected to a longitudinal loading T and pressure loading $p = p_0$ on the lateral surface.

Some relevant formulae

$$\epsilon_{rr} = \frac{\partial u_r}{\partial r}, \qquad \epsilon_{r\theta} = \frac{1}{2}\left[\frac{1}{r}\frac{\partial u_r}{\partial \theta} + r\frac{\partial}{\partial r}\left(\frac{u_\theta}{r}\right)\right],$$

$$\epsilon_{\theta\theta} = \frac{1}{r}\left(\frac{\partial u_\theta}{\partial \theta} + u_r\right), \qquad \epsilon_{\theta z} = \frac{1}{2}\left(\frac{\partial u_\theta}{\partial z} + \frac{1}{r}\frac{\partial u_z}{\partial \theta}\right),$$

$$\epsilon_{zz} = \frac{\partial u_z}{\partial z}, \qquad \epsilon_{rz} = \frac{1}{2}\left(\frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r}\right).$$

$$\boldsymbol{\tau} = \lambda(\mathrm{tr}\,\boldsymbol{\epsilon})\boldsymbol{I} + 2\mu\boldsymbol{\epsilon}.$$

where

$$\lambda = \frac{E\nu}{(1+\nu)(1-2\nu)}, \quad \mu = \frac{E}{2(1+\nu)}.$$