Indian Institute of Science
ME 242: Midsemester Test

Date: 3/10/13.
Duration: 3.45 p.m.–5.15 p.m.
Maximum Marks: 100

1. If $\boldsymbol{w}$ is the axial vector of a skew-symmetric tensor $\boldsymbol{W}$, and $\boldsymbol{T}$ is an arbitrary tensor, then first determine if $\boldsymbol{T}\boldsymbol{W}\boldsymbol{T}^T$ is a skew-symmetric tensor, and if it is, then find the axial vector of $\boldsymbol{T}\boldsymbol{W}\boldsymbol{T}^T$ in terms of $\mathbf{cof}\,\boldsymbol{T}$ and $\boldsymbol{w}$. By taking $\boldsymbol{T} = \boldsymbol{Q}$, where $\boldsymbol{Q}$ is proper orthogonal, deduce the axial vector of $\boldsymbol{Q}\boldsymbol{W}\boldsymbol{Q}^T$ in terms of $\boldsymbol{Q}$ and $\boldsymbol{w}$ (your final answer should be in terms of $\boldsymbol{Q}$ and $\boldsymbol{w}$, and *not* in terms of $\mathbf{cof}\,\boldsymbol{Q}$ and $\boldsymbol{w}$). State whatever formulae you use (without proving). (35)

2. Let $\boldsymbol{X}$ and $\boldsymbol{x}$ denote the position vectors of a point in the reference and deformed configurations. (40)

   (a) If $\boldsymbol{w}(\boldsymbol{x}, t)$ is a vector-valued spatial field, then *derive* the formula for the material derivative $D\boldsymbol{w}/Dt$ in the Eulerian setting.

   (b) If
   $$\boldsymbol{w} = \frac{\boldsymbol{x}}{|\boldsymbol{x}|},$$
   and if the Lagrangian description of velocity is given by
   $$\tilde{\boldsymbol{v}}(\boldsymbol{X}, t) = \boldsymbol{X}(1 + t),$$
   then *using the formula that you derived in part (a) above*, find the material derivative $D\boldsymbol{w}/Dt$ in the Eulerian setting, i.e., in terms of $(\boldsymbol{x}, t)$. (Hint: Note that $\boldsymbol{x}|_{t=0} = \boldsymbol{X}$).

3. A circular cylinder of radius a and length L is constrained along its axis, so that it can only deform along the radial direction when a pressure p is applied on its lateral surface as shown in Fig. 1. Assuming $u_r = cr$, $u_z = d_1 + d_2 z$, where c, d_1 and d_2 are constants that you have to determine (not guess), and making suitable assumptions about u_θ (justify without proving), find the displacement and stress fields in the circular cylinder. You may directly use the given formulae below. (25)

Some relevant formulae

$$\begin{aligned}
w_i &= -\frac{1}{2}\epsilon_{ijk}W_{jk}, \\
W_{ij} &= -\epsilon_{ijk}w_k, \\
(\mathbf{cof}\,\boldsymbol{T})_{ij} &= \frac{1}{2}\epsilon_{imn}\epsilon_{jpq}T_{mp}T_{nq}, \\
(\mathbf{cof}\,\boldsymbol{T})^T\boldsymbol{T} &= (\det \boldsymbol{T})\boldsymbol{I}.
\end{aligned}$$

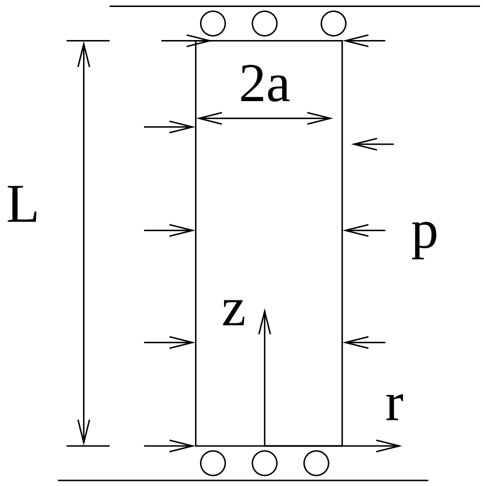

Figure 1: A circular cylinder constrained along its flat edges.

$$\epsilon_{rr} = \frac{\partial u_r}{\partial r}, \qquad \epsilon_{r\theta} = \frac{1}{2}\left[\frac{1}{r}\frac{\partial u_r}{\partial \theta} + r\frac{\partial}{\partial r}\left(\frac{u_\theta}{r}\right)\right],$$

$$\epsilon_{\theta\theta} = \frac{1}{r}\left(\frac{\partial u_\theta}{\partial \theta} + u_r\right), \qquad \epsilon_{\theta z} = \frac{1}{2}\left(\frac{\partial u_\theta}{\partial z} + \frac{1}{r}\frac{\partial u_z}{\partial \theta}\right),$$

$$\epsilon_{zz} = \frac{\partial u_z}{\partial z}, \qquad \epsilon_{rz} = \frac{1}{2}\left(\frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r}\right).$$

$$\boldsymbol{\tau} = \lambda(\mathrm{tr}\,\boldsymbol{\epsilon})\boldsymbol{I} + 2\mu\boldsymbol{\epsilon}.$$

$$\frac{\partial \tau_{rr}}{\partial r} + \frac{1}{r}\frac{\partial \tau_{r\theta}}{\partial \theta} + \frac{\partial \tau_{rz}}{\partial z} + \frac{\tau_{rr} - \tau_{\theta\theta}}{r} = 0,$$

$$\frac{\partial \tau_{\theta r}}{\partial r} + \frac{1}{r}\frac{\partial \tau_{\theta\theta}}{\partial \theta} + \frac{\partial \tau_{\theta z}}{\partial z} + \frac{\tau_{r\theta} + \tau_{\theta r}}{r} = 0,$$

$$\frac{\partial \tau_{zr}}{\partial r} + \frac{1}{r}\frac{\partial \tau_{z\theta}}{\partial \theta} + \frac{\partial \tau_{zz}}{\partial z} + \frac{\tau_{zr}}{r} = 0.$$