Indian Institute of Science
ME 242: Midsemester Test

Date: 27/9/14.
Duration: 9.30 a.m.–11.00 a.m.
Maximum Marks: 100

1. Let $\boldsymbol{S} = \sum_{i=1}^{3} \lambda_i \boldsymbol{e}_i^* \otimes \boldsymbol{e}_i^*$ be the spectral resolution of a symmetric tensor. Using the formula (25)

$$(\textbf{cof}\,\boldsymbol{T})^T = I_2 \boldsymbol{I} - I_1 \boldsymbol{T} + \boldsymbol{T}^2$$

where I_1 and I_2 are the first and second principal invariants of a second-order tensor $\boldsymbol{T}$, find the spectral resolution of $\textbf{cof}\,\boldsymbol{S}$. You may directly use the formulae for I_1 and I_2 in terms of the eigenvalues in case you remember them (or derive them if you don't). Use this spectral resolution to find the eigenvalues and the corresponding eigenvectors of $\textbf{cof}\,\boldsymbol{S}$.

2. In what follows, a subscript comma denotes differentiation, e.g., $\phi_{,i} = \partial\phi/\partial x_i$. A function ϕ is said to be harmonic if (40)

$$\boldsymbol{\nabla}^2 \phi = \phi_{,ii} = 0,$$

and is said to be biharmonic if

$$\boldsymbol{\nabla}^4 \phi := \boldsymbol{\nabla}^2(\boldsymbol{\nabla}^2 \phi) = 0.$$

Given that ϕ is harmonic, determine if $(\boldsymbol{x} \cdot \boldsymbol{x})\phi$ is biharmonic. You may use the comma notation.

3. Treat this problem as a two-dimensional problem by ignoring the z-coordinate, z-displacement etc. Thus, the deformation gradient, velocity gradient etc. are 2×2 matrices. A point with position vector (X, Y) at $t = 0$, occupies the position (x, y) after time t as shown in Fig. 1, by rotating through an angle ωt and moving radially outward by δR, where $R = \sqrt{X^2 + Y^2}$, and ω and δ are constants. (35)

   (a) Find the motion $\boldsymbol{\chi}(\boldsymbol{X}, t)$, the deformation gradient $\boldsymbol{F}$, and the Lagrangian and small strain tensors $\boldsymbol{E}$ and $\boldsymbol{\epsilon}$ at time t (Hint: Express your motion in terms of an orthogonal tensor; the subsequent calculations will be easier since you can use $\boldsymbol{Q}^{-1} = \boldsymbol{Q}^T$ etc.).

   (b) Find the Lagrangian and Eulerian velocities $(\tilde{\boldsymbol{v}}, \boldsymbol{v})$ and accelerations $(\tilde{\boldsymbol{a}}, \boldsymbol{a})$. The Eulerian acceleration $\boldsymbol{a}(\boldsymbol{x}, t)$ must be found using the expression for $\boldsymbol{v}(\boldsymbol{x}, t)$.

   (c) Find the body force $\boldsymbol{b}$ at the point (x, y), and the traction $\boldsymbol{t}$ at the point $(a\cos\alpha, a\sin\alpha)$ (in the reference configuration) shown in the figure, where a is the radius of the disc.

Some relevant formulae

$$\boldsymbol{\tau} = \lambda(\operatorname{tr}\boldsymbol{\epsilon})\boldsymbol{I} + 2\mu\boldsymbol{\epsilon},$$
$$\cos(A + B) = \cos A \cos B - \sin A \sin B,$$
$$\sin(A + B) = \sin A \cos B + \sin B \cos A.$$

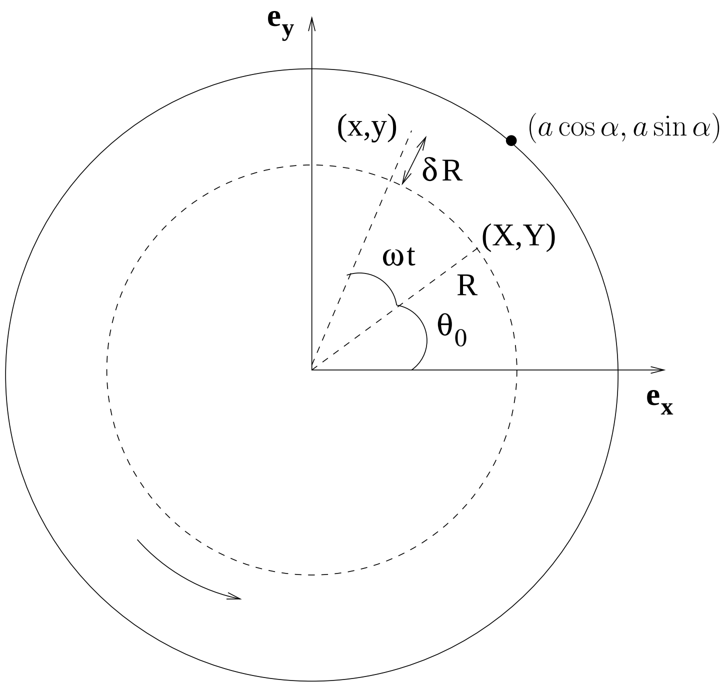

Figure 1: A spinning circular disc of radius a.