Indian Institute of Science
ME 242: Midsemester Test

Date: 7/11/14.
Duration: 10.00 a.m.–11.00 a.m.
Maximum Marks: 10

1. A circular disc of radius $a+\delta$ is shrink-fitted into a hole of radius a in an infinite domain of the same material as the disc by first cooling it, placing it into the hole, and then allowing the temperature to increase to room temperature (see Fig. 1). Assuming the Airy stress function to be given by

$$\phi^{(i)} = Ar^2 + B\log r + Cr^2\log r,$$
$$\phi^{(o)} = Pr^2 + Q\log r + Rr^2\log r,$$

in the inner and outer domains, and assuming plane stress conditions, find the displacement and stress fields in the disc and in the outer domain. Ignore thermal effects and assume $\delta \ll a$. While finding the displacements from the strain-displacement relations, you may make appropriate assumptions about the nature of the displacement field based on symmetry considerations. *State your assumptions clearly.* The relevant formulae for plane stress conditions are

$$\tau_{rr} = \frac{1}{r}\frac{\partial\phi}{\partial r} + \frac{1}{r^2}\frac{\partial^2\phi}{\partial\theta^2}, \qquad \epsilon_{rr} = \frac{\partial u_r}{\partial r},$$
$$\tau_{\theta\theta} = \frac{\partial^2\phi}{\partial r^2}, \qquad \epsilon_{\theta\theta} = \frac{1}{r}\left(\frac{\partial u_\theta}{\partial\theta} + u_r\right),$$
$$\tau_{r\theta} = -\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial\phi}{\partial\theta}\right), \qquad \epsilon_{r\theta} = \frac{1}{2}\left[\frac{1}{r}\frac{\partial u_r}{\partial\theta} + r\frac{\partial}{\partial r}\left(\frac{u_\theta}{r}\right)\right].$$

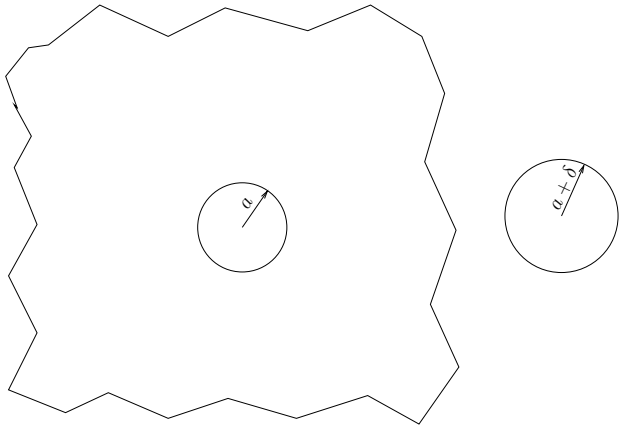

Figure 1: Circular disc of radius $a+\delta$ shrink-fitted into a hole of radius a in an infinite domain.