Indian Institute of Science
ME 242: Midsemester Test

Date: 28/9/19.
Duration: 9.30 a.m.–11.00 a.m.
Maximum Marks: 100

1. Using the formula (30)

$$\boldsymbol{u}\cdot(\boldsymbol{v}\times\boldsymbol{w}) = \epsilon_{ijk}u_i v_j w_k,$$

evaluate $\boldsymbol{a}\cdot(\boldsymbol{b}\times\boldsymbol{c})$ where $\boldsymbol{a}=\boldsymbol{u}\times\boldsymbol{v}$, $\boldsymbol{b}=\boldsymbol{v}\times\boldsymbol{w}$ and $\boldsymbol{c}=\boldsymbol{w}\times\boldsymbol{u}$ in terms of $\boldsymbol{u}\cdot(\boldsymbol{v}\times\boldsymbol{w})$.

2. Let $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$ be arbitrary vectors. (35)

   (a) Using indicial notation, determine whether

$$\begin{gathered}\boldsymbol{a}\cdot(\boldsymbol{b}\times\boldsymbol{c}) = \boldsymbol{b}\cdot(\boldsymbol{c}\times\boldsymbol{a}) = \boldsymbol{c}\cdot(\boldsymbol{a}\times\boldsymbol{b}),\\ \boldsymbol{a}\times(\boldsymbol{b}\times\boldsymbol{c}) = (\boldsymbol{a}\cdot\boldsymbol{c})\boldsymbol{b} - (\boldsymbol{a}\cdot\boldsymbol{b})\boldsymbol{c}.\end{gathered}$$

   (b) Let $g := \boldsymbol{a}\cdot(\boldsymbol{b}\times\boldsymbol{c})$, and let $W(g)$ be a function of g. It is given that

$$\boldsymbol{a}\times\frac{\partial W}{\partial \boldsymbol{a}} + \boldsymbol{b}\times\frac{\partial W}{\partial \boldsymbol{b}} + \boldsymbol{c}\times\frac{\partial W}{\partial \boldsymbol{c}}$$

   is a constant vector independent of $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$. Using the results from Part (a), find this constant vector.

3. Treat this problem as a two-dimensional problem by ignoring the z-coordinate, z-displacement (35) etc. Thus, the deformation gradient, velocity gradient etc. are 2×2 matrices. The motion in the $\bar{\boldsymbol{e}}_1$–$\bar{\boldsymbol{e}}_2$ coordinate system is given by

$$\begin{aligned}\bar{x} &= \bar{X}\cos\gamma t,\\ \bar{y} &= \bar{Y},\end{aligned}$$

where γ is a constant, and $(\bar{x},\bar{y})$ and $(\bar{X},\bar{Y})$ denote the deformed and undeformed coordinates with respect to the $\bar{\boldsymbol{e}}_1$-$\bar{\boldsymbol{e}}_2$ coordinate system. The $\bar{\boldsymbol{e}}_1$-$\bar{\boldsymbol{e}}_2$ coordinate system rotates about the $\boldsymbol{e}_3$ axis as shown with $\bar{\boldsymbol{e}}_1$ making an angle $\theta(t)$ with respect to the $\boldsymbol{e}_1$ axis. With respect to the $\boldsymbol{e}_1$-$\boldsymbol{e}_2$ coordinate system,

   (a) Find the motion $\boldsymbol{\chi}(\boldsymbol{X},t)$, the deformation gradient $\boldsymbol{F}$, and the Lagrangian and small strain tensors $\boldsymbol{E}$ and $\boldsymbol{\epsilon}$ at time t.

   (b) Find the Lagrangian and Eulerian velocities $(\tilde{\boldsymbol{v}},\boldsymbol{v})$ and accelerations $(\tilde{\boldsymbol{a}},\boldsymbol{a})$. The Eulerian acceleration $\boldsymbol{a}(\boldsymbol{x},t)$ must be found using the expression for $\boldsymbol{v}(\boldsymbol{x},t)$.

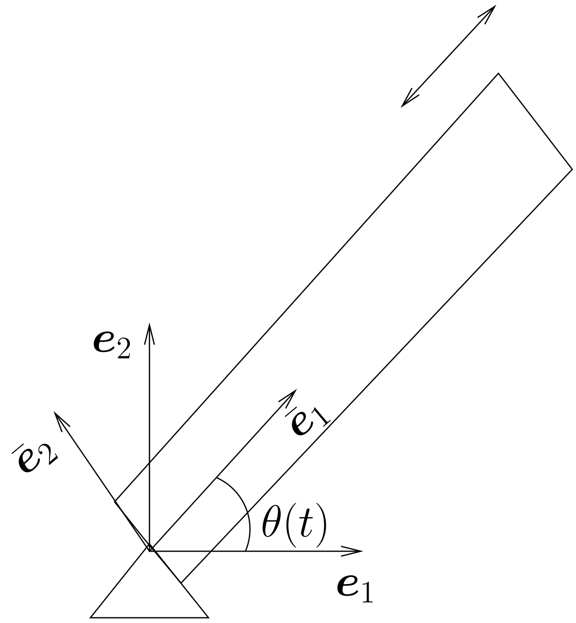

Figure 1: A spinning rod.