Indian Institute of Science
ME 242: Midsemester Test

Date: 2/11/19.
Duration: 9.30 a.m.–10.30 a.m.
Maximum Marks: 10

1. A disc of inner radius a, outer radius b, and unit width along the z-direction is subjected to equal and opposite moments M on the outer and inner boundaries $r = b$ and $r = a$ by the application of suitable tangential tractions which are independent of θ (see Fig. 1). The radial displacement is given by

$$u_r = c_1 \cos\theta + c_2 \sin\theta + \frac{c_3}{r},$$

where c_1, c_2 and c_3 are constants to be determined. Assuming $u_z = 0$, the *average* u_θ over the two-dimensional domain to be zero, and treating the problem as two-dimensional (i.e., $u_r = u_r(r,\theta)$ and $u_\theta = u_\theta(r,\theta)$), find the displacement components (u_r, u_θ) in terms of $(r, \theta, \lambda, \mu, M, a, b)$. Make reasonable assumptions and state them clearly. The governing equations are

$$\begin{aligned} 0 &= (\lambda+\mu)\frac{\partial(\operatorname{tr}\boldsymbol{\epsilon})}{\partial r} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial(ru_r)}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 u_r}{\partial\theta^2} - \frac{2}{r^2}\frac{\partial u_\theta}{\partial\theta}\right],\\ 0 &= (\lambda+\mu)\frac{1}{r}\frac{\partial(\operatorname{tr}\boldsymbol{\epsilon})}{\partial\theta} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial(ru_\theta)}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 u_\theta}{\partial\theta^2} + \frac{2}{r^2}\frac{\partial u_r}{\partial\theta}\right]. \end{aligned}$$

The strain-displacement and constitutive relations are

$$\epsilon_{rr} = \frac{\partial u_r}{\partial r}, \qquad \epsilon_{r\theta} = \frac{1}{2}\left[\frac{1}{r}\frac{\partial u_r}{\partial\theta} + r\frac{\partial}{\partial r}\left(\frac{u_\theta}{r}\right)\right],$$

$$\epsilon_{\theta\theta} = \frac{1}{r}\left(\frac{\partial u_\theta}{\partial\theta} + u_r\right), \qquad \operatorname{tr}\boldsymbol{\epsilon} = \epsilon_{rr} + \epsilon_{\theta\theta},$$

$$\boldsymbol{\tau} = \lambda(\operatorname{tr}\boldsymbol{\epsilon})\boldsymbol{I} + 2\mu\boldsymbol{\epsilon}.$$

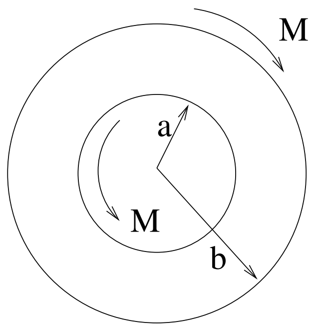

Figure 1: Hollow disk subjected to equal and opposite moments M on the outer and inner boundaries.