Indian Institute of Science
ME 242: Midsemester Test

Date: 13/11/21.
Duration: 9.30 a.m.–11.00 a.m.
Maximum Marks: 10

1. The wedge-shaped body shown in Fig. 1 with inner radius a, outer radius b, and included angle $2\theta_0$ is subjected to uniform pressure loading p_0 on the outer surface $r = b$, while the surface $r = a$ is traction free. The surfaces $\theta = \pm\theta_0$ have roller supports along them. The displacement field that satisfies the equations of equilibrium is given by

$$
\begin{aligned}
2\mu u_r &= 2c_1(1-2\nu)r - \frac{c_2}{r} - \frac{c_3\theta}{r} + c_4\cos\theta,\\
2\mu u_\theta &= -\frac{c_3\log r}{r} - c_4\sin\theta,
\end{aligned}
$$

where c_1, c_2, c_3 and c_4 are constants. The strain-displacement and constitutive relations are

$$
\begin{aligned}
\epsilon_{rr} &= \frac{\partial u_r}{\partial r}, & \epsilon_{r\theta} &= \frac{1}{2}\left[\frac{1}{r}\frac{\partial u_r}{\partial \theta} + r\frac{\partial}{\partial r}\left(\frac{u_\theta}{r}\right)\right],\\
\epsilon_{\theta\theta} &= \frac{1}{r}\left(\frac{\partial u_\theta}{\partial \theta} + u_r\right), & \operatorname{tr}\boldsymbol{\epsilon} &= \epsilon_{rr} + \epsilon_{\theta\theta},\\
\boldsymbol{\tau} &= \lambda(\operatorname{tr}\boldsymbol{\epsilon})\boldsymbol{I} + 2\mu\boldsymbol{\epsilon}.
\end{aligned}
$$

   (a) State the boundary conditions at the surfaces $r = a, b$ and $\theta = \pm\theta_0$.

   (b) Assuming plane strain conditions ($u_z = 0$), and treating the problem as two-dimensional (i.e., all field variables are functions of (r, θ) only), determine the constants c_1, c_2, c_3 and c_4 in terms of the given data. Every step should be *mathematically justified.*

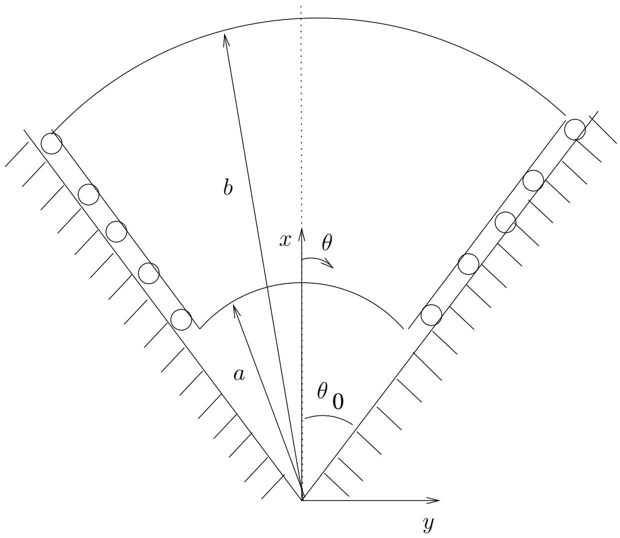

Figure 1: Constrained wedge with roller supports at $\theta = \pm\theta_0$ subjected to a uniform pressure loading at $r = b$, with the surface $r = a$ being traction free.