Indian Institute of Science
ME 242: Midsemester Test

Date: 1/10/22.
Duration: 9.30 a.m.–11.30 a.m.
Maximum Marks: 100

1. We have seen that a rotation can be written as (30)

$$\boldsymbol{R}(\boldsymbol{w}, \alpha) = \boldsymbol{I} + \sin\alpha\, \boldsymbol{W} + (1 - \cos\alpha)\boldsymbol{W}^2,$$

where $\boldsymbol{W} \in \text{Skw}$ whose axial vector $\boldsymbol{w}$ is of unit magnitude. Let $\boldsymbol{u}$ be a unit vector in the plane perpendicular to $\boldsymbol{w}$.

   (a) Determine $(\boldsymbol{u}, \boldsymbol{R}\boldsymbol{u})$ in terms of α. Derive any results that you require on the way.

   (b) Find the specific $\boldsymbol{R}$ (let us call it $\boldsymbol{R}_0$) in terms of $\boldsymbol{W}$ such that the axis of $\boldsymbol{R}_0$ is $\boldsymbol{w}$, and such that $\boldsymbol{R}_0\boldsymbol{u} = \boldsymbol{W}\boldsymbol{u}$.

2. Treat this problem as a two-dimensional problem by ignoring the z-coordinate, z-displacement etc. Thus, the deformation gradient, velocity gradient etc. are 2×2 matrices. Consider the rigid bar of length L and height h as shown in Fig. 1. At $t = 0$, the end A of the bar is at the origin of the $\boldsymbol{e}_1$-$\boldsymbol{e}_2$ coordinate system, the end B is at the corner of the two walls, and the bar is horizontal, i.e. $\theta(0) = 0$. At $t = 0$, the ends A and B of the centerline of the bar start sliding along the horizontal and vertical walls, respectively. Point A moves with a constant velocity v_0, so that at time t, the bar makes an angle $\theta(t)$ with respect to the horizontal. (40)

   (a) Consider the point that has coordinates (X, Y) at time $t = 0$ in the undeformed configuration. First find the coordinates of this point with respect to the $\tilde{\boldsymbol{e}}_1$-$\tilde{\boldsymbol{e}}_2$ coordinate system at time t.

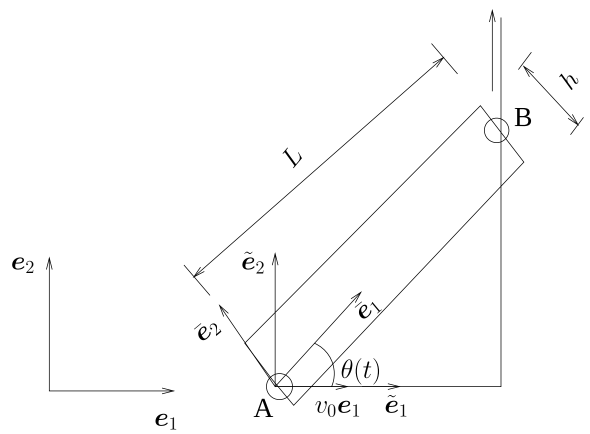

Figure 1: A bar with sliding ends.

(b) Next, find the coordinates of this point with respect to the $\boldsymbol{e}_1$-$\boldsymbol{e}_2$ coordinate system. This will determine the motion $\boldsymbol{\chi}(\boldsymbol{X}, t)$.

(c) Using this equation of motion, find the deformation gradient $\boldsymbol{F}$, and the Lagrangian and small strain tensors $\boldsymbol{E}$ and $\boldsymbol{\epsilon}$ at time t.

(d) Find the Lagrangian and Eulerian velocities $(\tilde{\boldsymbol{v}}, \boldsymbol{v})$, the velocity gradient $\boldsymbol{L}$, and the rate of deformation $\boldsymbol{D}$. Using the constraint that point B slides along the vertical wall, determine the differential equation for finding $\theta(t)$, and state the initial condition for solving this differential equation. Do not attempt to solve this differential equation. You may use matrices to present your results in a compact manner.

3. Let $\boldsymbol{A} \in \text{Lin}$ be an invertible *constant* tensor (not dependent on $\boldsymbol{x}$ or $\boldsymbol{x}^*$). Let (30)

$$\boldsymbol{x}^* = \boldsymbol{A}\boldsymbol{x},$$
$$\boldsymbol{T}^* = \boldsymbol{A}\boldsymbol{T}\boldsymbol{A}^T.$$

Find an expression for $\boldsymbol{\nabla}_{\boldsymbol{x}^*} \cdot \boldsymbol{T}^*$ (which denotes the divergence of $\boldsymbol{T}^*$ with respect to $\boldsymbol{x}^*$) in terms of $\boldsymbol{\nabla}_{\boldsymbol{x}} \cdot \boldsymbol{T}$ (which denotes the divergence of $\boldsymbol{T}$ with respect to $\boldsymbol{x}$) and $\boldsymbol{A}$.

Some relevant formulae

$$\boldsymbol{Q}_{ij} = \bar{\boldsymbol{e}}_i \cdot \boldsymbol{e}_j.$$