Indian Institute of Science
ME 242: Midsemester Test

Date: 6/11/22.
Duration: 9.30 a.m.–11.00 a.m.
Maximum Marks: 10

1. The boundary of a circular hole of radius a in an unbounded domain is subjected to a shear traction of the form $t_\theta = s_0 + s_1 \cos 2\theta + s_2 \sin 2\theta$, where s_0, s_1 and s_2 are given constants. The general solution under plane strain conditions is given by

$$
\begin{aligned}
2\mu u_r &= -\frac{F_1}{r} - \frac{\theta F_3}{r} + 2(1-2\nu) r\theta G_2 - \sum_{\substack{n=-\infty \\ n\neq 0,-1}}^{\infty} (n+1) r^n \Big\{ A_n \cos(n+1)\theta + B_n \sin(n+1)\theta \Big\} \\
&\quad - \sum_{\substack{n=-\infty \\ n\neq 0,1}}^{\infty} [n-3+4\nu]\, r^n \Big\{ C_n \cos(n-1)\theta + D_n \sin(n-1)\theta \Big\}, \\
2\mu u_\theta &= -\frac{F_2}{r} - \frac{\log r F_3}{r} - 4(1-\nu) r \log r G_2 \\
&\quad + \sum_{\substack{n=-\infty \\ n\neq 0,-1}}^{\infty} (n+1) r^n \Big\{ A_n \sin(n+1)\theta - B_n \cos(n+1)\theta \Big\} \\
&\quad + \sum_{\substack{n=-\infty \\ n\neq 0,1}}^{\infty} [n+3-4\nu]\, r^n \Big\{ C_n \sin(n-1)\theta - D_n \cos(n-1)\theta \Big\}, \\
\tau_{rr} &= \frac{F_1}{r^2} + \frac{\theta F_3}{r^2} + 2\theta G_2 - \sum_{\substack{n=-\infty \\ n\neq 0,-1}}^{\infty} n(n+1) r^{n-1} \Big\{ A_n \cos(n+1)\theta + B_n \sin(n+1)\theta \Big\} \\
&\quad - \sum_{\substack{n=-\infty \\ n\neq 0,1}}^{\infty} n(n-3) r^{n-1} \Big\{ C_n \cos(n-1)\theta + D_n \sin(n-1)\theta \Big\}, \\
\tau_{\theta\theta} &= -\frac{F_1}{r^2} - \frac{\theta F_3}{r^2} + 2\theta G_2 + \sum_{\substack{n=-\infty \\ n\neq 0,-1}}^{\infty} n(n+1) r^{n-1} \Big\{ A_n \cos(n+1)\theta + B_n \sin(n+1)\theta \Big\} \\
&\quad + \sum_{\substack{n=-\infty \\ n\neq 0,1}}^{\infty} n(n+1) r^{n-1} \Big\{ C_n \cos(n-1)\theta + D_n \sin(n-1)\theta \Big\}, \\
\tau_{r\theta} &= \frac{F_2}{r^2} + \frac{(\log r - 1) F_3}{r^2} - G_2 + \sum_{\substack{n=-\infty \\ n\neq 0,-1}}^{\infty} n(n+1) r^{n-1} \Big\{ A_n \sin(n+1)\theta - B_n \cos(n+1)\theta \Big\} \\
&\quad + \sum_{\substack{n=-\infty \\ n\neq 0,1}}^{\infty} n(n-1) r^{n-1} \Big\{ C_n \sin(n-1)\theta - D_n \cos(n-1)\theta \Big\}.
\end{aligned}
$$

Determine the expressions for the nonzero constants, and *state them clearly at the end of your solution.* If, for example, A_n corresponding to $n = -1$ is nonzero, then write it as A_{-1}. Give *justifications* for the constants that you claim are zero. Guesswork for the nonzero constants in the infinite series is fine provided these justifications are provided.