Indian Institute of Science
ME 242: Midsemester Test

Date: 7/11/25.
Duration: 3.00 p.m.–4.30 p.m.
Maximum Marks: 10

1. The boundary of a circular hole of radius a in an unbounded domain is subjected to a traction of the form $t_x = -s_0 \sin^3\theta$ and $t_y = s_0 \sin^2\theta \cos\theta$. The general solution under plane strain conditions is given by

$$
\begin{aligned}
2\mu u_r &= -\frac{F_1}{r} - \frac{\theta F_3}{r} + 2(1-2\nu) r\theta G_2 - \sum_{\substack{n=-\infty \\ n\neq 0,-1}}^{\infty} (n+1) r^n \Big\{ A_n \cos(n+1)\theta + B_n \sin(n+1)\theta \Big\} \\
&\quad - \sum_{\substack{n=-\infty \\ n\neq 0,1}}^{\infty} [n-3+4\nu]\, r^n \Big\{ C_n \cos(n-1)\theta + D_n \sin(n-1)\theta \Big\}, \\
2\mu u_\theta &= -\frac{F_2}{r} - \frac{\log r F_3}{r} - 4(1-\nu) r \log r G_2 \\
&\quad + \sum_{\substack{n=-\infty \\ n\neq 0,-1}}^{\infty} (n+1) r^n \Big\{ A_n \sin(n+1)\theta - B_n \cos(n+1)\theta \Big\} \\
&\quad + \sum_{\substack{n=-\infty \\ n\neq 0,1}}^{\infty} [n+3-4\nu]\, r^n \Big\{ C_n \sin(n-1)\theta - D_n \cos(n-1)\theta \Big\}, \\
\tau_{rr} &= \frac{F_1}{r^2} + \frac{\theta F_3}{r^2} + 2\theta G_2 - \sum_{\substack{n=-\infty \\ n\neq 0,-1}}^{\infty} n(n+1) r^{n-1} \Big\{ A_n \cos(n+1)\theta + B_n \sin(n+1)\theta \Big\} \\
&\quad - \sum_{\substack{n=-\infty \\ n\neq 0,1}}^{\infty} n(n-3) r^{n-1} \Big\{ C_n \cos(n-1)\theta + D_n \sin(n-1)\theta \Big\}, \\
\tau_{\theta\theta} &= -\frac{F_1}{r^2} - \frac{\theta F_3}{r^2} + 2\theta G_2 + \sum_{\substack{n=-\infty \\ n\neq 0,-1}}^{\infty} n(n+1) r^{n-1} \Big\{ A_n \cos(n+1)\theta + B_n \sin(n+1)\theta \Big\} \\
&\quad + \sum_{\substack{n=-\infty \\ n\neq 0,1}}^{\infty} n(n+1) r^{n-1} \Big\{ C_n \cos(n-1)\theta + D_n \sin(n-1)\theta \Big\}, \\
\tau_{r\theta} &= \frac{F_2}{r^2} + \frac{(\log r - 1) F_3}{r^2} - G_2 + \sum_{\substack{n=-\infty \\ n\neq 0,-1}}^{\infty} n(n+1) r^{n-1} \Big\{ A_n \sin(n+1)\theta - B_n \cos(n+1)\theta \Big\} \\
&\quad + \sum_{\substack{n=-\infty \\ n\neq 0,1}}^{\infty} n(n-1) r^{n-1} \Big\{ C_n \sin(n-1)\theta - D_n \cos(n-1)\theta \Big\}.
\end{aligned}
$$

Determine the expressions for the nonzero constants, and *state them clearly at the end of your solution.* If, for example, C_n corresponding to $n = -1$ is nonzero, then write it as C_{-1}. Give *justifications* for the constants that you claim are zero. Guesswork for the nonzero constants in the infinite series is fine provided these justifications are provided. Next find the resultant moment about the origin and the resultant force on the surface of the hole per unit length (into the paper).

Some relevant formulae

$$
\begin{aligned}
\sin(2\theta) &= 2\sin\theta\cos\theta, \\
\cos(2\theta) &= \cos^2\theta - \sin^2\theta, \\
\sin(3\theta) &= 3\sin\theta - 4\sin^3\theta, \\
\cos(3\theta) &= 4\cos^3\theta - 3\cos\theta.
\end{aligned}
$$