

Indian Institute of Science

ME 242: Midsemester Test

Date: 20/9/26.

Duration: 9.30 a.m.–11.30 a.m.

Maximum Marks: 100

1. If λ_i denote the eigenvalues of a positive definite symmetric tensor $\mathbf{S}$, determine $J_1 = \text{tr}(\mathbf{S}^{-1})$, $J_2 = \text{tr} \mathbf{S}$ and $J_3 = \text{tr}(\mathbf{S}^2)$ in terms of λ_i . It is proposed that if $\mathbf{S}$ is positive definite, then $J_i > 0$, $i = 1, 2, 3$. Conversely, it is proposed that if $J_i > 0$, $i = 1, 2, 3$, then $\mathbf{S}$ is positive definite. Determine if each of these statements is true. If either is not, then present a counterexample. (30)
2. Let a bounded domain be denoted by V and its surface by S . The governing equation for a vector field $\mathbf{u}$, which is a function of the position vector $\mathbf{x}$, is given by (35)

$$\lambda[\nabla \times (\nabla \times \mathbf{u})] - \mu \nabla(\nabla \cdot \mathbf{u}) = \mathbf{0},$$

where λ and μ are constants, and the ‘ ∇ ’ operator is with respect to $\mathbf{x}$. Take the dot product of the above equation with a vector $\mathbf{v}$ which is a function of $\mathbf{x}$, ‘integrate by parts’ over the domain V , and apply the divergence theorem to obtain an equation that is written in terms of $\int_V \dots dV$, where the integrand is in terms of $\nabla \times \mathbf{u}$, $\nabla \times \mathbf{v}$, $\nabla \cdot \mathbf{u}$ and $\nabla \cdot \mathbf{v}$, and a term of the form $\int_S \dots dS$. Using indicial notation, find a relation between $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$ and $\mathbf{b} \cdot (\mathbf{c} \times \mathbf{a})$, where $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ are arbitrary vectors. Use this relation to express one of the surface integrals in terms of $\mathbf{v} \times \mathbf{n}$. Write your final equation with the volume terms on the LHS, and the surface terms on the RHS. (Hint: Consider $\nabla \cdot [\mathbf{v} \times (\nabla \times \mathbf{u})]$).

3. Treat this entire problem as a two-dimensional problem so that all matrices such as deformation gradient, strain etc. are 2×2 matrices. A rigid link of length L is attached to fixed support at one end, and to a rectangular rigid panel at the other end using frictionless pin joints at both ends as shown in Fig. 1. A point $\mathbf{X} \equiv (X, Y)$ in the reference configuration gets mapped to the point $\mathbf{x} \equiv (x, y)$ in the deformed configuration as shown in the figure. Assume that the angles $\theta_1(t)$ and $\theta_2(t)$ are known functions of time t . Note that the position vectors $\mathbf{X}$ and $\mathbf{x}$ are measured with respect to the origin (at the intersection of $\mathbf{e}_1$ and $\mathbf{e}_2$) (35)
 - (a) Determine the mapping $\mathbf{x} = \boldsymbol{\chi}(\mathbf{X}, t)$. Write the $\boldsymbol{\chi}$ mapping in terms of matrices, e.g., $\boldsymbol{\chi} = \mathbf{A}\mathbf{X} + \mathbf{c}$. Answer the subsequent parts using this matrix notation.
 - (b) Determine $\mathbf{F}$, $\tilde{\mathbf{v}}(\mathbf{X}, t)$, $\mathbf{v}(\mathbf{x}, t)$, $\tilde{\mathbf{a}}(\mathbf{X}, t)$, $\mathbf{a}(\mathbf{x}, t)$ (this should be found using $\mathbf{v}(\mathbf{x}, t)$) and $\mathbf{D}$, where the symbols have the usual meanings.
 - (c) Find the Lagrangian strain $\mathbf{E}(\mathbf{X}, t)$ and the small strain tensor $\boldsymbol{\epsilon}(\mathbf{X}, t)$.

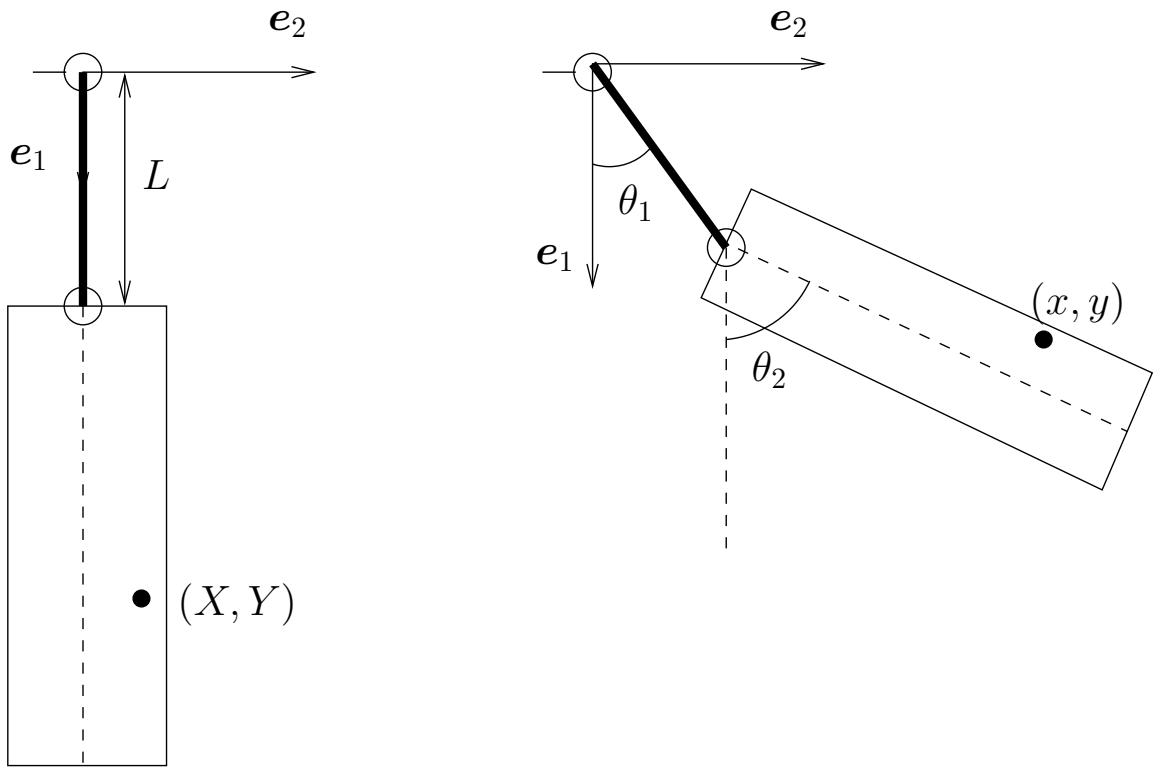


Figure 1: Reference and deformed configurations at times $t = 0$ and current time t .