Indian Institute of Science
ME 242: Midsemester Test

Date: 23/9/06.
Duration: 9.30 a.m.–11.00 p.m.
Maximum Marks: 100

Instructions:

1. Justify all your steps.
2. The points for each question are indicated in the right margin.

1. Let $\boldsymbol{W} \in$ Skw, and let $\boldsymbol{w}$ be its axial vector . By making a particular choice of $\boldsymbol{u}$ in the relation $\boldsymbol{W}\boldsymbol{u} = \boldsymbol{w} \times \boldsymbol{u}$, find using *indicial notation* a relation between the components W_{ij} and w_i. (25)

2. The body forces are said to be conservative if they are derivable from a scalar potential W_b, i.e., $\rho\boldsymbol{b} = \boldsymbol{\nabla}_x W_b$. Assuming ρ to be constant, show that the centrifugal body force $-\rho\boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \boldsymbol{x})$ is derivable from a potential of the form $W_b = \beta\rho(\boldsymbol{\Omega} \times \boldsymbol{x}) \cdot (\boldsymbol{\Omega} \times \boldsymbol{x})$, where β is a constant that you have to determine. You may directly use the result $\boldsymbol{u} \times (\boldsymbol{v} \times \boldsymbol{w}) = (\boldsymbol{u} \cdot \boldsymbol{w})\boldsymbol{v} - (\boldsymbol{u} \cdot \boldsymbol{v})\boldsymbol{w}$. (25)

3. We assume in this problem that the body forces are conservative, i.e., $\rho\boldsymbol{b} = \boldsymbol{\nabla}_x W_b$. The curl of a second-order tensor $\boldsymbol{T}$, denoted by $\boldsymbol{\nabla} \times \boldsymbol{T}$, is given by (50)

$$(\boldsymbol{\nabla} \times \boldsymbol{T})_{ij} = \epsilon_{irs} \frac{\partial T_{js}}{\partial x_r}.$$

   (a) If $\boldsymbol{H} \in$ Sym, find an expression for the components of $\boldsymbol{\nabla} \times (\boldsymbol{\nabla} \times \boldsymbol{H})$, and using this expression and the symmetry of $\boldsymbol{H}$, determine if $\boldsymbol{\nabla} \times (\boldsymbol{\nabla} \times \boldsymbol{H})$ is symmetric.

   (b) Evaluate the vector $\boldsymbol{\nabla} \cdot [\boldsymbol{\nabla} \times (\boldsymbol{\nabla} \times \boldsymbol{H})]$.

   (c) If $\boldsymbol{\tau}$ is taken to be $\boldsymbol{\nabla} \times (\boldsymbol{\nabla} \times \boldsymbol{H}) - W_b\boldsymbol{I}$, determine using the above results if the equilibrium equation given by $\boldsymbol{\nabla}_x \cdot \boldsymbol{\tau} + \boldsymbol{\nabla}_x W_b = \boldsymbol{0}$ is automatically satisfied.

   (d) Assume that there are no body forces ($W_b = 0$), and that $\boldsymbol{\tau} = \boldsymbol{\nabla} \times (\boldsymbol{\nabla} \times \boldsymbol{H})$. Let $H_{11} = -G\alpha xyz$, $H_{22} = G\alpha xyz$, with all the remaining components zero (G and α are constants). Find the stress field.

   (e) Assuming that this stress field exists in a circular cylinder with the z-axis coinciding with the axis of the cylinder, and with the x and y axes on the bottom surface, find the tractions on the lateral and top surface of the cylinder.

   (f) Does one have to check the conditions of compatibility in order to find the displacement field? Justify your answer. (Do not actually check the compatibility conditions even if your answer to this question is 'Yes')