Indian Institute of Science
ME 242: Midsemester Test

Date: 29/9/10.
Duration: 11.00 a.m.–12.30 p.m.
Maximum Marks: 100

Instructions:

1. Justify all your steps.
2. The points for each question are indicated in the right margin.

1. Let $\boldsymbol{w}_1$ and $\boldsymbol{w}_2$ be perpendicular vectors, and let $\boldsymbol{v} = \boldsymbol{w}_1 \times \boldsymbol{w}_2$. Determine mathematically if $\boldsymbol{v} \times (\boldsymbol{w}_1 + \boldsymbol{w}_2)$ and $\boldsymbol{v} \times (\boldsymbol{w}_1 - \boldsymbol{w}_2)$ are perpendicular to each other (*Derive* any results that you need on the way). If they are not perpendicular, find the condition under which they are perpendicular. (25)

2. Let $p(\boldsymbol{x}) = G(\boldsymbol{x}) \cos(k\,|\boldsymbol{x}|)$, where $\boldsymbol{x}$ is the position vector, $|\boldsymbol{x}| = \sqrt{\boldsymbol{x} \cdot \boldsymbol{x}}$, and $p(\boldsymbol{x})$ and $G(\boldsymbol{x})$ are scalar-valued field variables. If (40)

$$\boldsymbol{\nabla}^2 p + k^2 p = 0,$$

where k is a constant, and $\boldsymbol{\nabla} \equiv \boldsymbol{e}_i \partial()/\partial x_i$, find the governing equation for $G(\boldsymbol{x})$ (although you can use indicial notation, your final answer should be in tensorial form, and will involve G, $\boldsymbol{\nabla}^2 G$, and so on.)

3. A hollow cylinder of length L and inner and outer radii a and b, is fixed at $r = b$, and subjected to a uniform moment M, and uniform pressure p at the inner surface $r = a$, as shown in Fig. 1. By assuming the body forces to be zero, and the displacement field to be given by (35)

$$\begin{aligned} u_r &= c_1 r + \frac{c_2}{r}, \\ u_\theta &= c_3 r + \frac{c_4}{r}, \\ u_z &= 0, \end{aligned}$$

use the strain-displacement and stress-strain relations, and boundary conditions to find the constants c_1, c_2, c_3 and c_4 in terms of terms of the Lame constants, M, a and b. Note that you need *not* check the equilibrium equations since the given displacement field satisfies them.

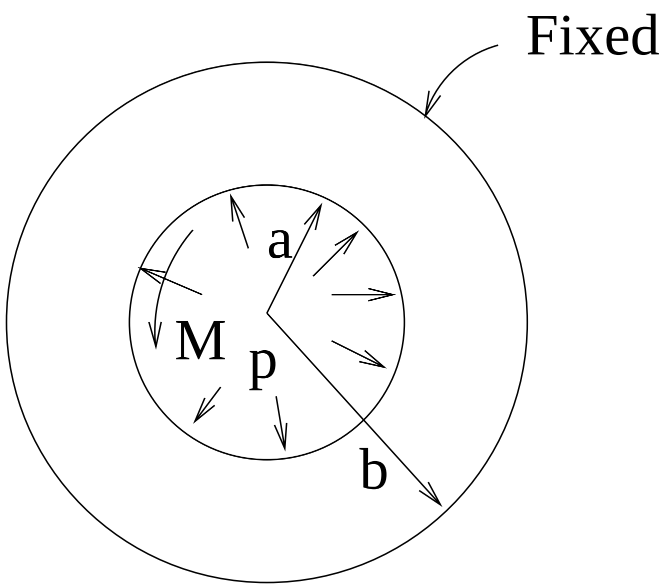

Figure 1: Hollow cylinder of length L fixed at the outer boundary $r = b$, and subjected to a moment M and pressure at the inner boundary $r = a$.

Some relevant formulae

$$\epsilon_{rr} = \frac{\partial u_r}{\partial r}, \qquad \epsilon_{r\theta} = \frac{1}{2}\left[\frac{1}{r}\frac{\partial u_r}{\partial \theta} + r\frac{\partial}{\partial r}\left(\frac{u_\theta}{r}\right)\right],$$

$$\epsilon_{\theta\theta} = \frac{1}{r}\left(\frac{\partial u_\theta}{\partial \theta} + u_r\right), \qquad \epsilon_{\theta z} = \frac{1}{2}\left(\frac{\partial u_\theta}{\partial z} + \frac{1}{r}\frac{\partial u_z}{\partial \theta}\right),$$

$$\epsilon_{zz} = \frac{\partial u_z}{\partial z}, \qquad \epsilon_{rz} = \frac{1}{2}\left(\frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r}\right).$$

$$\boldsymbol{\tau} = \lambda(\mathrm{tr}\,\boldsymbol{\epsilon})\boldsymbol{I} + 2\mu\boldsymbol{\epsilon}.$$