ME243: Assignment 1

Due: 20/8/15

1. Do the vectors $(1,1,1)$, $(-1,0,1)$ and $(1,1,0)$ form a basis for $\Re^3$? Justify.

2. Show that the vector product in the form of the determinant

$$\boldsymbol{u} \times \boldsymbol{v} = \begin{vmatrix} \boldsymbol{e}_1 & \boldsymbol{e}_2 & \boldsymbol{e}_3 \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$$

   is equivalent to the following indicial notation

$$\boldsymbol{u} \times \boldsymbol{v} = \epsilon_{ijk} u_j v_k \boldsymbol{e}_i$$

   (expand the determinant and the indicial notation and compare term by term to show the equivalence).

3. Evaluate $\epsilon_{ijk}\epsilon_{ijm}$.

4. Evaluate $\delta_{ik}\delta_{ik}$ and $\delta_{ik}\delta_{im}$.

5. Which of the following transformations are second-order tensors (justify)?
   (i) $\boldsymbol{T}(\boldsymbol{v}) = \alpha \boldsymbol{v}$ (α a scalar)
   (ii) $T(\boldsymbol{v}) = \sin(\boldsymbol{a} \cdot \boldsymbol{v})$
   (iii) $\boldsymbol{T}(\boldsymbol{v}) = \boldsymbol{a} \times \boldsymbol{v}$
   (iv) $\boldsymbol{T}(\boldsymbol{v}) = \boldsymbol{v} + \boldsymbol{a}$

6. Show that $\operatorname{tr} \boldsymbol{RS} = \operatorname{tr} \boldsymbol{SR} = \operatorname{tr} \boldsymbol{R}^T \boldsymbol{S}^T = \operatorname{tr} \boldsymbol{S}^T \boldsymbol{R}^T$.

7. If $\boldsymbol{S}$ and $\boldsymbol{W}$ are symmetric and antisymmetric tensors, respectively, and $\boldsymbol{T}$ is an arbitrary second-order tensor prove without using indicial notation that

$$\boldsymbol{S} : \boldsymbol{T} = \boldsymbol{S} : \boldsymbol{T}^T = \boldsymbol{S} : \left[\frac{1}{2}(\boldsymbol{T}^T + \boldsymbol{T})\right]$$

$$\boldsymbol{W} : \boldsymbol{T} = -\boldsymbol{W} : \boldsymbol{T}^T = \boldsymbol{W} : \left[\frac{1}{2}(\boldsymbol{T} - \boldsymbol{T}^T)\right]$$

$$\boldsymbol{S} : \boldsymbol{W} = 0.$$