

ME243: Assignment 10

Due: 3/11/15

1. Show that the response functions, $\hat{\mathbf{T}}_{s=0}^{\infty}$ and $\hat{\mathbf{S}}_{s=0}^{\infty}$ for the first and second Piola-Kirchhoff stress satisfy the axiom of material frame-indifference if and only

$$\hat{\mathbf{T}}_{s=0}^{\infty}(\mathbf{X}, \mathbf{Q}(t-s)\mathbf{F}(\mathbf{X}, t-s)) = \mathbf{Q}(t) \hat{\mathbf{T}}_{s=0}^{\infty}(\mathbf{X}, \mathbf{F}(\mathbf{X}, t-s)) \quad \forall \mathbf{F} \in \text{Lin}^+, \mathbf{Q} \in \text{Orth}^+,$$

$$\hat{\mathbf{S}}_{s=0}^{\infty}(\mathbf{X}, \mathbf{Q}(t-s)\mathbf{F}(\mathbf{X}, t-s)) = \hat{\mathbf{S}}_{s=0}^{\infty}(\mathbf{X}, \mathbf{F}(\mathbf{X}, t-s)) \quad \forall \mathbf{F} \in \text{Lin}^+, \mathbf{Q} \in \text{Orth}^+.$$

2. Show that the response functions, $\hat{\mathbf{T}}_{s=0}^{\infty}$ and $\hat{\mathbf{S}}_{s=0}^{\infty}$ for an isotropic material satisfy

$$\hat{\mathbf{T}}_{s=0}^{\infty}(\mathbf{X}, \mathbf{F}(\mathbf{X}, t-s)\mathbf{Q}(t-s)) = \hat{\mathbf{T}}_{s=0}^{\infty}(\mathbf{X}, \mathbf{F}(\mathbf{X}, t-s))\mathbf{Q}(t) \quad \forall \mathbf{F} \in \text{Lin}^+, \mathbf{Q} \in \text{Orth}^+,$$

$$\hat{\mathbf{S}}_{s=0}^{\infty}(\mathbf{X}, \mathbf{F}(\mathbf{X}, t-s)\mathbf{Q}(t-s)) = \mathbf{Q}^T(t) \hat{\mathbf{S}}_{s=0}^{\infty}(\mathbf{X}, \mathbf{F}(\mathbf{X}, t-s))\mathbf{Q}(t) \quad \forall \mathbf{F} \in \text{Lin}^+, \mathbf{Q} \in \text{Orth}^+.$$

3. The constitutive relation for a body is given by

$$\boldsymbol{\tau}(\mathbf{x}, t) = \alpha[\mathbf{F}(\mathbf{X}, t)\mathbf{e}] \otimes [\mathbf{F}(\mathbf{X}, t)\mathbf{e}],$$

where \mathbf{e} is a unit vector and α is a constant which does not depend on position or time. Is the material

- elastic?
- homogeneous?
- frame-indifferent?
- isotropic?