ME243: Assignment 11

Due: 10/11/15

1. Consider a homogeneous, isotropic material subjected to simple shear. Show that the Cauchy stress tensor components τ_{13} and τ_{23} are zero, and that τ_{12} is an odd function of γ. Also show that

   $$\tau_{11} - \tau_{22} = \gamma \tau_{12}.$$

   Note that the above equation is valid regardless of the material properties of the body; it is called Rivlin's universal relation. It shows that

   $$\tau_{11} \neq \tau_{22} \text{ for } \gamma \neq 0.$$

   which is known as the *Poynting effect*.

2. A rubber cylinder with radius R and length L in its natural state is rotated about its axis of symmety with constant angular speed ω, the motion being given by

   $$\begin{aligned} x_1 &= \frac{1}{\sqrt{\lambda}}(X_1 \cos\omega t - X_2 \sin\omega t), \\ x_2 &= \frac{1}{\sqrt{\lambda}}(X_1 \sin\omega t + X_2 \cos\omega t), \\ x_3 &= \lambda X_3, \end{aligned}$$

   where λ is a positive constant (to be determined). Is the motion isochoric? The rubber is incompressible and may be regarded as a *Mooney* material characterized by the constitutive relation

   $$\boldsymbol{\tau} = -p\boldsymbol{I} + (\alpha + \beta \operatorname{tr} \boldsymbol{B})\boldsymbol{B} - \beta \boldsymbol{B}^2,$$

   where α and β are positive constants. Assuming that the curved boundary of the cylinder is traction-free and that no body forces act find the pressure p. Assuming further that the resultant forces on the end-faces are zero, obtain an equation from which the length of the spinning cylinder may be obtained.

3. A ball of radius R is made of foam rubber whose constitutive relation is given by

   $$\boldsymbol{\tau} = (\det \boldsymbol{B})^{-3/2} \left[(\psi(\det \boldsymbol{B}) - \beta \operatorname{tr}(\mathbf{cof}\, \boldsymbol{B}))\boldsymbol{I} + (\alpha \det \boldsymbol{B} + \beta \operatorname{tr} \boldsymbol{B})\boldsymbol{B} - \beta \boldsymbol{B}^2 \right],$$

   where $\psi(\det \boldsymbol{B}) = -\alpha\sqrt{\det \boldsymbol{B}} + \beta(\det \boldsymbol{B})^{3/2}$. If the ball is subjected to a homeogeneous compression, $\boldsymbol{x} = \boldsymbol{X}/\gamma$, by the application of a uniform pressure P, show that the radius is reduced to R/γ, where

   $$(\gamma^5 - 1)(\alpha\gamma + \beta) = P.$$