ME243: Assignment 12

Due: 17/11/15

1. Show that the integral form of the mechanical energy balance in the reference configuration for a hyperelastic material can be written as

$$\frac{d}{dt}\int_{V_0}\left(\rho_0\frac{\dot{\boldsymbol{\chi}}\cdot\dot{\boldsymbol{\chi}}}{2}+\hat{W}(\boldsymbol{X},\boldsymbol{F}(\boldsymbol{X},t))\right)dV=\int_{S_0}\dot{\boldsymbol{\chi}}\cdot(\boldsymbol{T}\boldsymbol{n}^0)\,dS+\int_{V_0}\dot{\boldsymbol{\chi}}\cdot\boldsymbol{f}^0\,dV,$$

where $\hat{W}(\boldsymbol{X},\boldsymbol{F}(\boldsymbol{X},t))$ is the stored energy function. Note that if $\boldsymbol{f}^0=\boldsymbol{0}$, and if $(\boldsymbol{T}\boldsymbol{n}^0)\cdot\dot{\boldsymbol{\chi}}=0$ on S_0, then the total energy is conserved, i.e.,

$$\int_{V_0}\left(\rho_0\frac{\dot{\boldsymbol{\chi}}\cdot\dot{\boldsymbol{\chi}}}{2}+\hat{W}(\boldsymbol{X},\boldsymbol{F}(\boldsymbol{X},t))\right)dV=\text{constant}.$$

2. Let there be a given a homogeneous, isotropic, hyperelastic material, whose reference configuration is a natural state, and whose stored energy function is given by

$$\check{W}(\boldsymbol{E})=\frac{\lambda}{2}(\operatorname{tr}\boldsymbol{E})^2+\mu\operatorname{tr}(\boldsymbol{E}^2)+\alpha_1(\operatorname{tr}\boldsymbol{E})^3+\alpha_2(\operatorname{tr}\boldsymbol{E})\operatorname{tr}(\boldsymbol{E}^2)+\alpha_3\operatorname{tr}(\boldsymbol{E}^3),$$

where α_1, α_2 and α_3 are constants. Find the associated response function $\check{\boldsymbol{S}}(\boldsymbol{E})$. This model which in essence is one step further than a St Venant-Kirchoff material (since it includes second-order terms in the constitutive relation for $\boldsymbol{S}$).

3. Show that

$$D\hat{\boldsymbol{T}}(\boldsymbol{F})[\boldsymbol{W}\boldsymbol{F}]=\boldsymbol{W}\hat{\boldsymbol{T}}(\boldsymbol{F}).$$

4. Relate $D\hat{\boldsymbol{\tau}}(\boldsymbol{I})[\boldsymbol{U}]$ and $D\hat{\boldsymbol{T}}(\boldsymbol{I})[\boldsymbol{U}]$ when $\hat{\boldsymbol{\tau}}(\boldsymbol{I})=\hat{\boldsymbol{T}}(\boldsymbol{I})\neq\boldsymbol{0}$.