ME243: Assignment 13

Due: 24/11/15

1. An incompressible Riener-Rivlin fluid whose constitutive relation is given by

$$\boldsymbol{\tau} = -p\boldsymbol{I} + \beta_1((I_2)_{\boldsymbol{D}}, (I_3)_{\boldsymbol{D}})\boldsymbol{D} + \beta_2((I_2)_{\boldsymbol{D}}, (I_3)_{\boldsymbol{D}})\boldsymbol{D}^2,$$

   fills the space between parallel rigid plates. The lower plate is held fixed and a steady shearing motion is produced by translating the other plate in its own plane with constant speed V. Introducing a Cartesian coordinate system in which the stationary and moving plates occupy the planes $x_2 = 0$ and $x_2 = d$ respectively, and the moving plate travels in the 1-direction, and assuming that the velocity field in the fluid is of the form $v_1 = v_1(x_2)$, $v_2 = 0$ and $v_3 = 0$, calculate the stress components. Verify that the stress component τ_{12} is a function of D_{12} only. Given that there is no pressure gradient in the 1-direction, β_1 and β_2 are constants and that no body forces act, find the velocity and stress distributions. Compare your results with the solution for a Newtonian fluid.

2. Show that

$$\begin{aligned}\boldsymbol{\epsilon} : \boldsymbol{\epsilon} + \boldsymbol{W} : \boldsymbol{W} &= \boldsymbol{\nabla}\boldsymbol{u} : (\boldsymbol{\nabla}\boldsymbol{u}).\\ \boldsymbol{\epsilon} : \boldsymbol{\epsilon} - \boldsymbol{W} : \boldsymbol{W} &= \boldsymbol{\nabla}\boldsymbol{u} : (\boldsymbol{\nabla}\boldsymbol{u})^T.\end{aligned}$$

3. Let $\boldsymbol{u} = \boldsymbol{0}$ on the surface S of a volume V. Derive the relation

$$\boldsymbol{\nabla}\boldsymbol{u} : (\boldsymbol{\nabla}\boldsymbol{u})^T = \boldsymbol{\nabla} \cdot ((\boldsymbol{\nabla}\boldsymbol{u})\boldsymbol{u} - (\boldsymbol{\nabla} \cdot \boldsymbol{u})\boldsymbol{u}) + (\boldsymbol{\nabla} \cdot \boldsymbol{u})^2,$$

   and use it to prove *Korn's inequality*:

$$\int_V \boldsymbol{\nabla}\boldsymbol{u} : \boldsymbol{\nabla}\boldsymbol{u}\, dV \leq 2\int_V \boldsymbol{\epsilon} : \boldsymbol{\epsilon}\, dV.$$

4. Show that

$$\begin{aligned}D\boldsymbol{U}(\boldsymbol{I})[\boldsymbol{\nabla}\boldsymbol{u}] = D\boldsymbol{V}(\boldsymbol{I})[\boldsymbol{\nabla}\boldsymbol{u}] &= \boldsymbol{\epsilon},\\ DJ(\boldsymbol{I})[\boldsymbol{\nabla}\boldsymbol{u}] = \boldsymbol{\nabla} \cdot \boldsymbol{u} &= \operatorname{tr}\boldsymbol{\epsilon}.\end{aligned}$$

   where $J = \det\boldsymbol{F}$, and $\boldsymbol{U}$ and $\boldsymbol{V}$ are the tensors which occur in the polar decomposition of $\boldsymbol{F}$.

5. Assuming that the residual stress vanishes, show that

$$\hat{\boldsymbol{\tau}}(\boldsymbol{F}) = \mathsf{C}[\boldsymbol{\epsilon}] + \mathrm{o}\,(\boldsymbol{\nabla}\boldsymbol{u})\,.$$

6. Assuming that the residual stress, $\hat{\boldsymbol{T}}(\boldsymbol{I})$, does not vanish, show that

$$\hat{\boldsymbol{T}}(\boldsymbol{F}) = \hat{\boldsymbol{T}}(\boldsymbol{I}) + \mathsf{C}[\boldsymbol{\epsilon}] + \boldsymbol{W}\hat{\boldsymbol{T}}(\boldsymbol{I}) + \mathrm{o}\,(\boldsymbol{\nabla}\boldsymbol{u})\,,$$

   where $\boldsymbol{\epsilon}$ and $\boldsymbol{W}$ are the symmetric and skew-symmetric parts of $\boldsymbol{\nabla}\boldsymbol{u}$, respectively.