ME243: Assignment 2

Due: 28/8/14

1. Prove using indicial notation that

$$\det(\boldsymbol{RS}) = \det(\boldsymbol{R})\det(\boldsymbol{S})$$

2. Using the relation $\mathbf{cof}\,\boldsymbol{T} = (\det\boldsymbol{T})\boldsymbol{T}^{-T}$ for an invertible $\boldsymbol{T}$, compute the eigenvalues of $\mathbf{cof}\,\boldsymbol{T}$ in terms of the eigenvalues of $\boldsymbol{T}$ (denoted by λ_1, λ_2 and λ_3). (Hint: First find the eigenvalues of $(\mathbf{cof}\,\boldsymbol{T})^T$, and then use the fact that $\det\boldsymbol{S} = \det\boldsymbol{S}^T$.)

3. Show that if $\boldsymbol{R}$ and $\boldsymbol{S}$ are second-order tensors, then

$$\begin{aligned}\det\boldsymbol{R} &= \frac{1}{3}\boldsymbol{R} : \mathbf{cof}\,\boldsymbol{R},\\ \det(\boldsymbol{R}+\boldsymbol{S}) &= \det\boldsymbol{R} + \mathbf{cof}\,\boldsymbol{R} : \boldsymbol{S} + \boldsymbol{R} : \mathbf{cof}\,\boldsymbol{S} + \det\boldsymbol{S}.\end{aligned}$$

4. Using *direct notation* show that (also verify using indicial notation)

$$\begin{aligned}(\boldsymbol{a}\otimes\boldsymbol{b})^T &= \boldsymbol{b}\otimes\boldsymbol{a},\\ (\boldsymbol{a}\otimes\boldsymbol{b})(\boldsymbol{c}\otimes\boldsymbol{d}) &= (\boldsymbol{b}\cdot\boldsymbol{c})\boldsymbol{a}\otimes\boldsymbol{d},\\ (\boldsymbol{a}\otimes\boldsymbol{b}) : (\boldsymbol{u}\otimes\boldsymbol{v}) &= (\boldsymbol{a}\cdot\boldsymbol{u})(\boldsymbol{b}\cdot\boldsymbol{v}),\\ \boldsymbol{T}(\boldsymbol{a}\otimes\boldsymbol{b}) &= (\boldsymbol{T}\boldsymbol{a})\otimes\boldsymbol{b},\\ (\boldsymbol{a}\otimes\boldsymbol{b})\boldsymbol{T} &= \boldsymbol{a}\otimes(\boldsymbol{T}^T\boldsymbol{b}).\end{aligned}$$

   Use any of the above results to show that for a symmetric tensor, $\boldsymbol{S}$, the spectral decompositions of $\boldsymbol{S}^2$ and $\boldsymbol{S}^{-1}$ (when it exists) are

$$\begin{aligned}\boldsymbol{S}^2 &= \lambda_1^2\boldsymbol{e}_1^*\otimes\boldsymbol{e}_1^* + \lambda_2^2\boldsymbol{e}_2^*\otimes\boldsymbol{e}_2^* + \lambda_3^2\boldsymbol{e}_3^*\otimes\boldsymbol{e}_3^*,\\ \boldsymbol{S}^{-1} &= \lambda_1^{-1}\boldsymbol{e}_1^*\otimes\boldsymbol{e}_1^* + \lambda_2^{-1}\boldsymbol{e}_2^*\otimes\boldsymbol{e}_2^* + \lambda_3^{-1}\boldsymbol{e}_3^*\otimes\boldsymbol{e}_3^*.\end{aligned}$$

5. Show that

$$\begin{aligned}&u_iv_jw_k\left\{[\boldsymbol{T}\boldsymbol{e}_i,\boldsymbol{e}_j,\boldsymbol{e}_k] + [\boldsymbol{e}_i,\boldsymbol{T}\boldsymbol{e}_j,\boldsymbol{e}_k] + [\boldsymbol{e}_i,\boldsymbol{e}_j,\boldsymbol{T}\boldsymbol{e}_k]\right\} =\\ &\qquad\epsilon_{ijk}u_iv_jw_k\left\{[\boldsymbol{T}\boldsymbol{e}_1,\boldsymbol{e}_2,\boldsymbol{e}_3] + [\boldsymbol{e}_1,\boldsymbol{T}\boldsymbol{e}_2,\boldsymbol{e}_3] + [\boldsymbol{e}_1,\boldsymbol{e}_2,\boldsymbol{T}\boldsymbol{e}_3]\right\}\end{aligned}$$