ME243: Assignment 3

Due: 3/9/2015

1. Using the spectral resolution, prove the Cayley-Hamilton theorem for symmetric second-order tensors.

2. Show that a (not necessarily symmetric) tensor $\boldsymbol{T}$ commutes with every skew tensor $\boldsymbol{W}$ if and only if $\boldsymbol{T} = \lambda \boldsymbol{I}$.

3. Find the principal values and principal directions (*without* using the results of theorem) of the tensor whose matrix representation in a particular basis is
$$\boldsymbol{S} = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 1 & -3\sqrt{3} \\ 0 & -3\sqrt{3} & -5 \end{bmatrix}.$$

4. Let $\boldsymbol{Q} \in \text{Orth}$. Show that if $\boldsymbol{n}$ is an eigenvector of a symmetric tensor $\boldsymbol{S}$, then $\boldsymbol{Q}\boldsymbol{n}$ is an eigenvector of $\boldsymbol{Q}\boldsymbol{S}\boldsymbol{Q}^T$ corresponding to the same eigenvalue.

5. Let $\boldsymbol{F} = \boldsymbol{R}\boldsymbol{U} = \boldsymbol{V}\boldsymbol{R}$ denote the polar decomposition of $\boldsymbol{F}$.
(i) Show that $\boldsymbol{U}$ and $\boldsymbol{V}$ have the same eigenvalues.
(ii) If $\boldsymbol{e}_i$ and $\boldsymbol{f}_i$ represent the eigenvectors of $\boldsymbol{U}$ and $\boldsymbol{V}$ corresponding to the eigenvalue λ_i, show that $\boldsymbol{F}$ and $\boldsymbol{R}$ can be represented as
$$\boldsymbol{F} = \sum_i \lambda_i \boldsymbol{f}_i \otimes \boldsymbol{e}_i,$$
$$\boldsymbol{R} = \sum_i \boldsymbol{f}_i \otimes \boldsymbol{e}_i.$$

6. Show that $\boldsymbol{S}^n$, $\boldsymbol{S}^{-1}$ and $\lambda(\operatorname{tr} \boldsymbol{S})\boldsymbol{I} + 2\mu\boldsymbol{S}$ are isotropic functions.