ME243: Assignment 4

Due: 10/9/15

1. Show that for any orthogonal $\boldsymbol{Q}(t)$, the tensor $\dot{\boldsymbol{Q}}\boldsymbol{Q}^T$ is skew at each time t.

2. Find the directional derivatives of the following functions

$$
\begin{aligned}
\boldsymbol{G}(\boldsymbol{T}) &= \boldsymbol{T}^3, \\
\boldsymbol{G}(\boldsymbol{T}) &= (\operatorname{tr} \boldsymbol{T})\boldsymbol{T}, \\
\boldsymbol{G}(\boldsymbol{T}) &= \boldsymbol{T}\boldsymbol{S}\boldsymbol{T}, \quad (\boldsymbol{S} \text{ given}) \\
\boldsymbol{G}(\boldsymbol{T}) &= \boldsymbol{T}^T\boldsymbol{T}, \\
\boldsymbol{G}(\boldsymbol{T}) &= (\boldsymbol{u} \cdot \boldsymbol{T}\boldsymbol{u})\boldsymbol{T}, \quad (\boldsymbol{u} \text{ given}) \\
\boldsymbol{G}(\boldsymbol{T}) &= \det(\boldsymbol{T}^2), \\
\boldsymbol{G}(\boldsymbol{v}) &= e^{\boldsymbol{v}\cdot\boldsymbol{v}}.
\end{aligned}
$$

3. (a) $\boldsymbol{\nabla} \times \boldsymbol{\nabla}\phi = \boldsymbol{0}$; (b) $\boldsymbol{\nabla} \cdot (\boldsymbol{\nabla} \times \boldsymbol{u}) = 0$.

4. $\boldsymbol{\nabla} \times (\boldsymbol{u} \times \boldsymbol{v}) = (\boldsymbol{\nabla} \cdot \boldsymbol{v})\boldsymbol{u} - (\boldsymbol{\nabla}\boldsymbol{v})\boldsymbol{u} - (\boldsymbol{\nabla} \cdot \boldsymbol{u})\boldsymbol{v} + (\boldsymbol{\nabla}\boldsymbol{u})\boldsymbol{v}$.

5. Show that

$$
(\boldsymbol{\nabla}\boldsymbol{u})\boldsymbol{u} = \frac{1}{2}\boldsymbol{\nabla}(\boldsymbol{u} \cdot \boldsymbol{u}) - \boldsymbol{u} \times (\boldsymbol{\nabla} \times \boldsymbol{u}).
$$

6. Let $\boldsymbol{v} = \boldsymbol{\nabla} \times \boldsymbol{u}$. Use the above results to show that

$$
\boldsymbol{\nabla} \times ((\boldsymbol{\nabla}\boldsymbol{u})\boldsymbol{u}) = (\boldsymbol{\nabla} \cdot \boldsymbol{u})\boldsymbol{v} + (\boldsymbol{\nabla}\boldsymbol{v})\boldsymbol{u} - (\boldsymbol{\nabla}\boldsymbol{u})\boldsymbol{v}.
$$

7. Let $\boldsymbol{W} = \frac{1}{2}\left(\boldsymbol{\nabla}\boldsymbol{u} - (\boldsymbol{\nabla}\boldsymbol{u})^T\right)$, and let $\boldsymbol{\omega} = \boldsymbol{\nabla} \times \boldsymbol{u}$. Then show that

$$
\begin{aligned}
W_{ij} &= -\frac{1}{2}\epsilon_{ijk}\omega_k, \\
\omega_i &= -\epsilon_{ijk}W_{jk}.
\end{aligned}
$$

   Thus, $\boldsymbol{\omega}$ is twice the axial vector of $\boldsymbol{W}$.

8. Evaluate $(\boldsymbol{\nabla}\boldsymbol{u})\boldsymbol{u}$ where $\boldsymbol{u} = (\boldsymbol{x}/|\boldsymbol{x}|)$.