ME243: Assignment 5

Due: 24/9/15

1. Show that a material area element dS_0 remains unchanged in area if and only if $\boldsymbol{F} = \boldsymbol{Q}$, where $\boldsymbol{Q} \in \text{Orth}^+$.

2. Show that

$$\frac{\partial J}{\partial \boldsymbol{C}} = \frac{1}{2} J \boldsymbol{C}^{-1},$$
$$\frac{\partial J}{\partial \boldsymbol{E}} = J \boldsymbol{C}^{-1}.$$

3. Consider the following deformations corresponding to pure extension, rigid body motion, and pure shear, respectively:

$$x_1 = (1+\epsilon)X_1;\; x_2 = (1-\nu\epsilon)X_2;\; x_3 = (1-\nu\epsilon)X_3,$$
$$\boldsymbol{x} = \boldsymbol{Q}(t)\boldsymbol{X} + \boldsymbol{a}(t),$$
$$x_1 = X_1 + \gamma X_2;\; x_2 = X_2;\; x_3 = X_3.$$

   Evaluate the tensors, $\boldsymbol{F}$, $\boldsymbol{C}$, $\boldsymbol{B}$, $\boldsymbol{E}$, $\bar{\boldsymbol{E}}$. Which of the deformations are volume preserving? Evaluate the small strain tensor $((\boldsymbol{\nabla u}+(\boldsymbol{\nabla u})^T)/2)$ for the rigid body motion case. Is it zero?

4. Show that

$$(I_1)_{\boldsymbol{C}} = 3 + 2(I_1)_{\boldsymbol{E}},$$
$$(I_2)_{\boldsymbol{C}} = 3 + 4(I_1)_{\boldsymbol{E}} + 4(I_2)_{\boldsymbol{E}},$$
$$(I_3)_{\boldsymbol{C}} = 1 + 2(I_1)_{\boldsymbol{E}} + 4(I_2)_{\boldsymbol{E}} + 8(I_3)_{\boldsymbol{E}}.$$