# ME243: Assignment 6

Due: 29/9/15

1. If $\boldsymbol{u} = \boldsymbol{x}/\left|\boldsymbol{x}\right|^3$, prove that $\boldsymbol{\nabla} \times \boldsymbol{u} = \boldsymbol{0}$ and $\boldsymbol{\nabla} \cdot \boldsymbol{u} = 0$. Find $\phi$ such that $\boldsymbol{u} = \boldsymbol{\nabla}\phi$. ($\phi$ which satisfies $\boldsymbol{\nabla}^2\phi = 0$ is said to be *harmonic*).

2. Show that the acceleration field of a rigid motion has the form

$$\boldsymbol{a}(\boldsymbol{x}_1, t) = \boldsymbol{a}(\boldsymbol{x}_2, t) + \dot{\boldsymbol{w}}(t) \times (\boldsymbol{x}_1 - \boldsymbol{x}_2) + \boldsymbol{w}(t) \times [\boldsymbol{w}(t) \times (\boldsymbol{x}_1 - \boldsymbol{x}_2)],$$

   where $\boldsymbol{w}(t)$ is the angular velocity.

3. Show that

$$\boldsymbol{\nabla} \cdot ((\boldsymbol{\nabla v})\boldsymbol{v}) = \boldsymbol{\nabla v} : \boldsymbol{\nabla v}^T + \boldsymbol{v} \cdot (\boldsymbol{\nabla}(\boldsymbol{\nabla} \cdot \boldsymbol{v})).$$

4. Using the definition of the material derivative

$$\frac{D\boldsymbol{v}}{Dt} = \frac{\partial \boldsymbol{v}}{\partial t} + (\boldsymbol{\nabla v})\boldsymbol{v},$$

   show that

$$\boldsymbol{\nabla} \cdot \left(\frac{D\boldsymbol{v}}{Dt}\right) = \frac{D}{Dt}(\boldsymbol{\nabla} \cdot \boldsymbol{v}) + \boldsymbol{D} : \boldsymbol{D} - \boldsymbol{W} : \boldsymbol{W}.$$

5. Consider the two-dimensional flow field given by $v_x = 1/(1+t)$, $v_y = 2t$, $v_z = 0$. Find the mapping $\boldsymbol{\chi}$ for a particle whose initial position is $\boldsymbol{X} = (1, 1, 0)$.

6. Given the velocity field

$$\boldsymbol{v} = (t^2 + 5t)\boldsymbol{e}_x + (y^2 - z^2 - 1)\boldsymbol{e}_y - (y^2 + 2yz)\boldsymbol{e}_z,$$

   compute at $t = 2$ and $\boldsymbol{x} = (3, 2, 4)$,

   (a) the acceleration (any approach will do);
   (b) the dilatation, i.e., $\operatorname{tr} \boldsymbol{D}$;
   (c) the vorticity.