ME243: Assignment 7

Due: 6/10/15

1. Prove the following transport theorems ($\boldsymbol{\lambda}$ represents the unit tangent vector to the contour C):

$$\frac{d}{dt}\oint_C f\,ds = \oint_C \left(\frac{Df}{Dt} + f\boldsymbol{\lambda}\cdot\boldsymbol{D}\boldsymbol{\lambda}\right) ds,$$

$$\frac{d}{dt}\int_S \boldsymbol{f}\cdot\boldsymbol{n}\,dS = \int_S \left(\frac{\partial \boldsymbol{f}}{\partial t} + (\boldsymbol{\nabla}\cdot\boldsymbol{f})\boldsymbol{v}\right)\cdot\boldsymbol{n}\,dS + \oint_C (\boldsymbol{f}\times\boldsymbol{v})\cdot\boldsymbol{\lambda}\,ds,$$

$$\frac{d}{dt}\int_S \boldsymbol{H}\boldsymbol{n}\,dS = \int_S \left[\frac{D\boldsymbol{H}}{Dt} + (\boldsymbol{\nabla}\cdot\boldsymbol{v})\boldsymbol{H} - \boldsymbol{H}(\boldsymbol{\nabla}\boldsymbol{v})^T\right]\boldsymbol{n}\,dS.$$

2. If $\boldsymbol{W}$ is the vorticity tensor and $\boldsymbol{\omega}$ is the vorticity vector, show that

$$\int_V \left[2\boldsymbol{W}\boldsymbol{v} + (\boldsymbol{\nabla}\cdot\boldsymbol{v})\boldsymbol{v}\right] dV = \int_S \left[(\boldsymbol{v}\cdot\boldsymbol{n})\boldsymbol{v} - \frac{\boldsymbol{v}\cdot\boldsymbol{v}}{2}\boldsymbol{n}\right] dS,$$

$$\boldsymbol{\nabla}\times\frac{D\boldsymbol{v}}{Dt} = \frac{D\boldsymbol{\omega}}{Dt} + (\boldsymbol{\nabla}\cdot\boldsymbol{v})\boldsymbol{\omega} - (\boldsymbol{\nabla}\boldsymbol{v})\boldsymbol{\omega}.$$

3. If $\boldsymbol{x}$ is the position vector of a point, and $\hat{\boldsymbol{t}}$ is the axial vector of $(\boldsymbol{T}-\boldsymbol{T}^T)$, show that

$$\int_V \left[\boldsymbol{x}\times(\boldsymbol{\nabla}\cdot\boldsymbol{T}) + \hat{\boldsymbol{t}}\right] dV = \int_S \boldsymbol{x}\times(\boldsymbol{T}\boldsymbol{n})\,dS.$$