ME243: Assignment 8

Due: 15/12/20

1. The stress tensor at the origin in a fluid is given by

$$\boldsymbol{\tau} = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 1 & -3\sqrt{3} \\ 0 & -3\sqrt{3} & -5 \end{bmatrix}$$

   Let the unit vector $\boldsymbol{n}$ have components:

$$\boldsymbol{n} = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta).$$

   Show that the normal component of stress on a surface whose outer normal is $\boldsymbol{n}$ is given by:

$$\tau_n = 4 - 3(\sin\theta\sin\phi + \sqrt{3}\cos\theta)^2.$$

   Hence find the directions of the normals to surfaces such that the normal stress is a maximum or a minimum. Compare these results with those obtained by solving the eigenvalue problem.

2. Show that the integral form of the mechanical energy balance in the reference configuration can be written as

$$\frac{d}{dt}\int_{V_0} \rho_0 \frac{\dot{\boldsymbol{\chi}}\cdot\dot{\boldsymbol{\chi}}}{2}\,dV = \int_{S_0} \dot{\boldsymbol{\chi}}\cdot(\boldsymbol{T}\boldsymbol{n}^0)\,dS + \int_{V_0} \rho_0\dot{\boldsymbol{\chi}}\cdot\boldsymbol{b}^0\,dV - \int_{V_0} \boldsymbol{T}:\dot{\boldsymbol{F}}\,dV.$$

3. The traction vectors on three planes at a point are

$$\begin{aligned} \boldsymbol{t}(\boldsymbol{n}) &= \boldsymbol{e}_1 + 2\boldsymbol{e}_2 + 3\boldsymbol{e}_3 \quad \text{for } \boldsymbol{n} = \boldsymbol{e}_1, \\ \boldsymbol{t}(\boldsymbol{n}) &= 2\sqrt{3}\boldsymbol{e}_1 + 2\sqrt{3}\boldsymbol{e}_2 \quad \text{for } \boldsymbol{n} = \frac{1}{\sqrt{3}}(\boldsymbol{e}_1 + \boldsymbol{e}_2 + \boldsymbol{e}_3), \\ \boldsymbol{t}(\boldsymbol{n}) &= 2(\boldsymbol{e}_1 + \boldsymbol{e}_2 + \boldsymbol{e}_3) \quad \text{for } \boldsymbol{n} = \boldsymbol{e}_2. \end{aligned}$$

   Find the stress tensor components with respect to the canonical basis.

4. Let $\boldsymbol{t}(\boldsymbol{n})$ and $\boldsymbol{t}(\boldsymbol{n}')$ be the traction vectors at a point on two plane elements with unit normals $\boldsymbol{n}$ and $\boldsymbol{n}'$. Find the traction vector on the plane containing $\boldsymbol{t}(\boldsymbol{n})$ and $\boldsymbol{t}(\boldsymbol{n}')$.

5. The nonzero stress components in a cylindrical bar of radius R and axis x_3 are given by $\tau_{13} = \tau_{31} = -\mu\alpha x_2$, $\tau_{23} = \tau_{32} = \mu\alpha x_1$, where μ and α are constants. Find the surface tractions on the surface of the cylinder.