ME243: Assignment 9

Due: 27/10/15

1. Prove that if $\boldsymbol{dx}_1$ and $\boldsymbol{dx}_2$ are two arbitrary elemental vectors moving with the body then

$$\frac{D}{Dt}(\boldsymbol{dx}_1 \cdot \boldsymbol{\tau}\boldsymbol{dx}_2) = \boldsymbol{dx}_1 \cdot \boldsymbol{\tau}^\diamond \boldsymbol{dx}_2,$$

   where $\boldsymbol{\tau}^\diamond$ is the convective stress rate.

2. Evaluate if the following function is frame-indifferent:

$$f = \boldsymbol{E}_d : \boldsymbol{E}_d - \alpha, \quad \alpha > 0,$$

   where $\boldsymbol{E}_d = \boldsymbol{E} - \frac{1}{3}(\mathrm{tr}\,\boldsymbol{E})\boldsymbol{I}$.

3. For each of the following constitutive relations, find if the principle of material frame-indifference is satisfied ($\boldsymbol{\tau}$ and $\boldsymbol{S}$ are Cauchy and second P-K stress tensors, respectively).

$$\begin{aligned}
\boldsymbol{\tau} &= -p(\rho, \theta)\boldsymbol{I}, \\
\boldsymbol{\tau} &= \alpha(\boldsymbol{F} + \boldsymbol{F}^T), \\
\boldsymbol{\tau} &= \beta_0(\theta, \rho, \mathcal{I}_{\boldsymbol{D}})\boldsymbol{I} + \beta_1(\theta, \rho, \mathcal{I}_{\boldsymbol{D}})\boldsymbol{D} + \beta_2(\theta, \rho, \mathcal{I}_{\boldsymbol{D}})\boldsymbol{D}^2, \\
\boldsymbol{S} &= \lambda(\mathrm{tr}\,\boldsymbol{E})\boldsymbol{I} + 2\mu\boldsymbol{E}.
\end{aligned}$$

4. We have seen that the second-Piola Kirchhoff stress remains unchanged under a change of observer, i.e., $\boldsymbol{S}^* = \boldsymbol{S}$ provided we postulate that $\boldsymbol{\tau}$ is frame-indifferent. If the heat flux vector over the reference configuration is defined as

$$\boldsymbol{q}^0(\boldsymbol{X}) = J\boldsymbol{F}^{-1}\boldsymbol{q}(\boldsymbol{x}),$$

   find the relation between $(\boldsymbol{q}^0)^*$ and $\boldsymbol{q}^0$ assuming that $\boldsymbol{q}$ is frame-indifferent.