Indian Institute of Science, Bangalore

ME 243: Endsemester Exam

Date: 9/12/10.
Duration: 9.30 a.m.–12.30 p.m.
Maximum Marks: 100

Instructions:
You may directly use any transport and divergence theorems needed, and formulae at the back. All questions carry 20 marks.

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1. If $\boldsymbol{w}$ is the axial vector of $\boldsymbol{W} \in \text{Skw}$, find an expression for $\mathbf{cof}\,\boldsymbol{W}$ in terms of $\boldsymbol{w}$. Deduce an expression for $e^{\mathbf{cof}\,\boldsymbol{W}}$ (the final expression should have finite number of terms) and its determinant.

2. The goal of this exercise is to find if the mechanical power $\int_{V(t)} \rho \boldsymbol{b} \cdot \boldsymbol{v}\, dV + \int_{S(t)} \boldsymbol{t} \cdot \boldsymbol{v}\, dS$ is frame-indifferent or objective.

   (a) Starting from the linear momentum balance equation (Eulerian form), derive the differential form of the mechanical energy balance.

   (b) Integrate this over a material volume and find the integral form of the mechanical energy balance.

   (c) Use this integral form to deduce if the mechanical power is objective. You may assume the stress and rate of deformation tensors to be objective, and velocity and acceleration to be non-objective. If the mechanical power is not objective, what term can you add or subtract from it in order to make it objective?

3. Consider a rigid body. State (without proving, but by quoting appropriate theorem) what simplification results in the integral form of the mechanical energy balance that you have derived in the preceding question. If further, $\int_{S(t)} \boldsymbol{t} \cdot \boldsymbol{v}\, dS = 0$, and the body force $\boldsymbol{b}$ is constant, deduce what quantity gets conserved, i.e., remains constant at all times. For the pendulum problem shown in Fig. 1, deduce if this 'conservation law' holds with respect to the fixed coordinate system x^*-y^* (where the constant body force is that of gravity), and if it does, use it to find the governing equation for the angle θ made by the pendulum. Do not attempt to solve this governing equation. Determine the constant in this governing equation by using the initial condition that the pendulum is released from rest at $\theta = \pi/2$. You may treat the pendulum as a one-dimensional object, i.e., $\rho\, dV \equiv m\, dx$ (Hint: You should solve the problem in the x^*-y^* frame of reference, where $\boldsymbol{b}$ is a constant, but should evaluate the integrals in the x-y system by writing $\boldsymbol{x}^* = (x \cos\theta, x \sin\theta)$.)

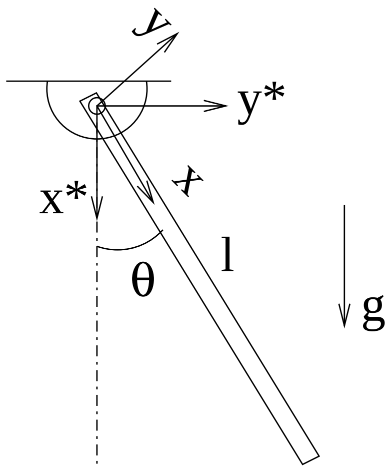

Figure 1: Oscillating rod.

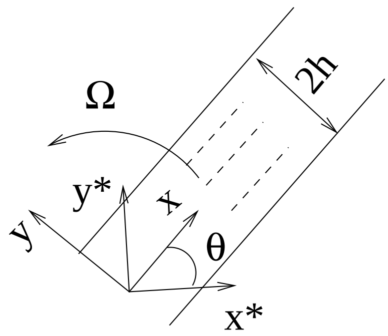

Figure 2: Fluid flowing in a rotating channel.

4. An incompressible fluid whose constitutive relation is given by $\boldsymbol{\tau} = -p\boldsymbol{I} + 2\mu\boldsymbol{D}$, where $\boldsymbol{D}$ is the rate of deformation tensor flows through a rotating channel shown in Fig. 2. Ω is *not* a constant. Assuming the flow to be two-dimensional (i.e., ignoring v_z and body forces in the x^*-y^* coordinate system), write the governing equations and boundary conditions for the velocity field (v_x, v_y) with respect to the rotating x-y coordinate system. Do not attempt to solve these equations.

5. A circular cylinder of radius a and length L is heated uniformly by T_Δ. Assuming that the axis of the cylinder remains fixed during the heating, assuming that the cylinder is free to expand, and assuming that the outer surface of the cylinder remains traction free, make an intelligent guess for the deformation $(r, \theta, z) = \boldsymbol{\chi}(R, \Theta, Z)$, and solve for the deformation and stresses. The constitutive relation is $\boldsymbol{S} = \lambda(\operatorname{tr}\boldsymbol{E})\boldsymbol{I} + 2\mu\boldsymbol{E} - (3\lambda + 2\mu)\alpha T_\Delta \boldsymbol{I}$, where α is the coefficient of thermal expansion.

Some relevant formulae

$$(\mathbf{cof}\,\boldsymbol{T})_{ij} = \frac{1}{2}\epsilon_{imn}\epsilon_{jpq}T_{mp}T_{nq},$$

$$\boldsymbol{\tau} = \frac{1}{J}\boldsymbol{F}\boldsymbol{S}\boldsymbol{F}^T,$$

$$w_i = -\frac{1}{2}\epsilon_{ijk}W_{jk},$$

$$W_{ij} = -\epsilon_{ijk}w_k,$$

$$\boldsymbol{b} = \boldsymbol{Q}^T\left[\boldsymbol{b}^* - \ddot{\boldsymbol{c}}\right] - \dot{\boldsymbol{\Omega}} \times \boldsymbol{x} - \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \boldsymbol{x}) - 2\boldsymbol{\Omega} \times \boldsymbol{v}.$$

$$\boldsymbol{\Omega} = \begin{bmatrix} \dot{\boldsymbol{e}}_2 \cdot \boldsymbol{e}_3 \\ \dot{\boldsymbol{e}}_3 \cdot \boldsymbol{e}_1 \\ \dot{\boldsymbol{e}}_1 \cdot \boldsymbol{e}_2 \end{bmatrix},$$

$$\boldsymbol{v}_2 - \boldsymbol{v}_1 = \boldsymbol{\omega} \times (\boldsymbol{x}_2 - \boldsymbol{x}_1),$$

$$F_{iJ} = \frac{h_i}{h_J}\frac{\partial \hat{\chi}_i}{\partial \eta_J}, \quad \text{no sum on } i,\ J, \quad h_i \equiv (1, r, 1), \quad h_J \equiv (1, R, 1).$$

If $\boldsymbol{\tau}$ is symmetric tensor-valued field, then the components of $\boldsymbol{\nabla}_x \cdot \boldsymbol{\tau}$ with respect to a cylindrical coordinate system are

$$\begin{aligned}
(\boldsymbol{\nabla} \cdot \boldsymbol{\tau})_r &= \frac{\partial \tau_{rr}}{\partial r} + \frac{1}{r}\frac{\partial \tau_{r\theta}}{\partial \theta} + \frac{\partial \tau_{rz}}{\partial z} + \frac{\tau_{rr} - \tau_{\theta\theta}}{r}, \\
(\boldsymbol{\nabla} \cdot \boldsymbol{\tau})_\theta &= \frac{\partial \tau_{\theta r}}{\partial r} + \frac{1}{r}\frac{\partial \tau_{\theta\theta}}{\partial \theta} + \frac{\partial \tau_{\theta z}}{\partial z} + \frac{2\tau_{r\theta}}{r}, \\
(\boldsymbol{\nabla} \cdot \boldsymbol{\tau})_z &= \frac{\partial \tau_{zr}}{\partial r} + \frac{1}{r}\frac{\partial \tau_{z\theta}}{\partial \theta} + \frac{\partial \tau_{zz}}{\partial z} + \frac{\tau_{zr}}{r}.
\end{aligned}$$