Indian Institute of Science, Bangalore

ME 243: Endsemester Exam

Date: 12/12/03.
Duration: 2.30 p.m.–5.30 p.m.
Maximum Marks: 100

1. Let (25)

$$V_\lambda = \{\boldsymbol{v} : \ \boldsymbol{S}\boldsymbol{v} = \lambda\boldsymbol{v}\}$$

be the characteristic space of $\boldsymbol{S} \in \mathrm{Sym}$ corresponding to the eigenvalue λ. Show that $\boldsymbol{T} \in \mathrm{Lin}$ commutes with $\boldsymbol{S}$ if and only if $\boldsymbol{T}$ maps every characteristic space of $\boldsymbol{S}$ into itself (i.e., if $\boldsymbol{e}_i^*$ is an eigenvector of $\boldsymbol{S}$ corresponding to the eigenvalue λ, then $\boldsymbol{T}\boldsymbol{e}_i^*$ is also an eigenvector of $\boldsymbol{S}$ corresponding to the same eigenvalue). (Hint: Orthonormal eigenvectors $\boldsymbol{e}_1^*$, $\boldsymbol{e}_2^*$, $\boldsymbol{e}_3^*$ constitute a basis for $\Re^3$.)

2. Using the relation $\partial J/\partial \boldsymbol{F} = \mathbf{cof}\,\boldsymbol{F}$, show that (25)

$$\begin{aligned}
&\boldsymbol{\nabla}_{\boldsymbol{X}} J = J\boldsymbol{\nabla}_{\boldsymbol{x}} \cdot \boldsymbol{F}^t, \\
&\boldsymbol{\nabla}_{\boldsymbol{x}} \cdot (J^{-1}\boldsymbol{F}^t) = \mathbf{0}, \\
&\frac{DJ}{Dt} = J\boldsymbol{\nabla}_{\boldsymbol{x}} \cdot \boldsymbol{v}.
\end{aligned}$$

(Hint: Use the first relation to prove the second.)

3. Let $\hat{\boldsymbol{\tau}}^*$ and $\hat{\boldsymbol{\tau}}$ represent the constitutive relations for $\boldsymbol{\tau}^*$ and $\boldsymbol{\tau}$, respectively, i.e., $\boldsymbol{\tau}^* = \hat{\boldsymbol{\tau}}^*(\boldsymbol{l}^*)$, and $\boldsymbol{\tau} = \hat{\boldsymbol{\tau}}(\boldsymbol{l})$, where $\boldsymbol{l}$ is a list of kinematical parameters on which $\boldsymbol{\tau}$ is assumed to depend. (20)

   (a) State *mathematically* the conditions of form invariance and invariance under superposed rigid-body motions for the above constitutive relation. Show that these conditions taken together imply observer-invariance.

   (b) For each of the constitutive relations

$$\begin{aligned}
\hat{\boldsymbol{\tau}}(\boldsymbol{F}) &= \beta_0\boldsymbol{I} + \beta_1\boldsymbol{B} + \beta_2(\boldsymbol{F}\boldsymbol{n}) \otimes (\boldsymbol{F}\boldsymbol{n}), \\
\hat{\boldsymbol{\tau}}(\boldsymbol{F}) &= \beta_0\boldsymbol{I} + \beta_1\boldsymbol{B} + \beta_2(\boldsymbol{B}\boldsymbol{e}) \otimes (\boldsymbol{B}\boldsymbol{e}),
\end{aligned}$$

   where β_0, β_1, β_2 are objective scalar parameters, $\boldsymbol{B} = \boldsymbol{F}\boldsymbol{F}^t$, $\boldsymbol{n}$ is a fixed vector in the reference configuration ($\boldsymbol{n}^* = \boldsymbol{n}$), and $\boldsymbol{e}$ is a fixed vector in the unstarred frame ($\boldsymbol{e}^* = \boldsymbol{Q}\boldsymbol{e}$), deduce which of the three conditions (observer-invariance, form invariance and invariance under a superposed rigid-body motion) are satisfied, and hence deduce which of them satisfies the principle of material frame-indifference.

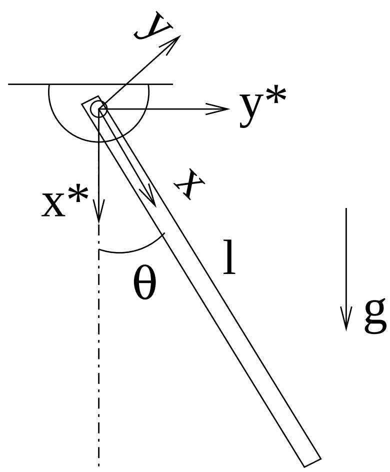

Figure 1:

4. A rod of length l and mass per unit length m, which is pinned at one end, oscillates about the x^* axis as shown in Fig. 1. Using the transformation law for the body force field given by (30)

$$\boldsymbol{b} = \boldsymbol{Q}^t \left[\boldsymbol{b}^* - \ddot{\boldsymbol{c}}\right] - \dot{\boldsymbol{\Omega}} \times \boldsymbol{x} - \boldsymbol{\Omega} \times \boldsymbol{\Omega} \times \boldsymbol{x} - 2\boldsymbol{\Omega} \times \boldsymbol{v},$$

where $\boldsymbol{\Omega}$ is the axial vector of $\boldsymbol{Q}^t\dot{\boldsymbol{Q}}$ with $Q_{ij} = \boldsymbol{e}_i^* \cdot \boldsymbol{e}_j$, find the body force field in the x-y-z frame. Then apply the principles of linear and angular momenta in integral form in the x-y-z frame, and find the governing equation for the angle θ, and the reactions exerted on the rod at the pinned-support with respect to the x-y-z axes. You may approximate the rod as a one-dimensional rigid body ($\rho dV \equiv m\,dx$), and neglect the effects of atmospheric pressure. Linearize the governing equation for θ, and present an explicit solution to this linearized equation.