Indian Institute of Science, Bangalore

ME 243: Endsemester Exam

Date: 12/12/07.
Duration: 2.30 p.m.–5.30 p.m.
Maximum Marks: 100

1. Without assuming that $\boldsymbol{A}$ and $\boldsymbol{B}$ commute, find a relation between $\det(e^{\boldsymbol{A}}e^{\boldsymbol{B}})$ and $\det(e^{\boldsymbol{A}+\boldsymbol{B}})$. (10)

2. If $\boldsymbol{T} \in \text{Lin}$ then show that (30)
$$\left|\boldsymbol{T}^T\boldsymbol{T}\right| \leq |\boldsymbol{T}|^2.$$

3. We have seen that in the absence of body forces and tractions on the entire surface, the linear and angular momentum of a body are conserved, i.e., if $\boldsymbol{b} = \boldsymbol{t} = \boldsymbol{0}$, then $\int_V \rho\boldsymbol{v}\, dV = \text{constant}$ and $\int_V \rho\boldsymbol{x}\times\boldsymbol{v}\, dV = \text{constant}$. Consider a body of arbitrary cross section and constant height h under plane strain conditions, with in-plane tractions and body forces zero, i.e., $v_z = 0$ in V, $t_x = t_y = 0$ on the entire surface S (but $t_z \neq 0$ on the top and bottom surfaces), and $\rho = \rho(x, y)$, $v_x = v_x(x, y)$ and $v_y = v_y(x, y)$. Determine if the linear and angular momenta of the body are conserved. (Hint: For some cases, you may need to write dV as $dz\, dA$). Also find $\int_S t_z\, dS$, $\int_S xt_z\, dS$ and $\int_S yt_z\, dS$. (35)

4. A circular shaft of initial radius, length and density R_0, L, and ρ_0 rotates with a steady-state angular speed of ω about the z-axis under plane-strain conditions. The lateral surfaces are traction free. The constitutive relation is given by (25)
$$\boldsymbol{\tau} = \mu(\boldsymbol{B} - \boldsymbol{I}),$$
where μ is a constant. Assume the deformation to be of the form
$$\begin{aligned} r &= f(R), \\ \theta &= \Theta, \\ z &= Z. \end{aligned}$$
Using the scale factors $h_i = (1, r, 1)$ and $h_J^0 = (1, R, 1)$, and the relation
$$F_{iJ} = \frac{h_i}{h_J^0}\frac{\partial\hat{\chi}_i}{\partial\eta_J}, \quad \text{no sum on } i,\ J,$$
where η_J denote (R, Θ, Z), find the deformation gradient. By first finding the body force using the formula $\boldsymbol{b} = -\boldsymbol{\Omega}\times(\boldsymbol{\Omega}\times\boldsymbol{x})$, and considering the equations of equilibrium in the *deformed* configuration, find the equation (*but do not solve*) and boundary conditions for determining the function $f(R)$.

Some relevant formulae

If $\boldsymbol{T}$ is tensor-valued field, then the components of $\boldsymbol{\nabla}_x \cdot \boldsymbol{T}$ with respect to a cylindrical coordinate system are

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\begin{aligned}
(\boldsymbol{\nabla} \cdot \boldsymbol{T})_r &= \frac{\partial T_{rr}}{\partial r} + \frac{1}{r}\frac{\partial T_{r\theta}}{\partial \theta} + \frac{\partial T_{rz}}{\partial z} + \frac{T_{rr} - T_{\theta\theta}}{r}, \\
(\boldsymbol{\nabla} \cdot \boldsymbol{T})_\theta &= \frac{\partial T_{\theta r}}{\partial r} + \frac{1}{r}\frac{\partial T_{\theta\theta}}{\partial \theta} + \frac{\partial T_{\theta z}}{\partial z} + \frac{T_{r\theta} + T_{\theta r}}{r}, \\
(\boldsymbol{\nabla} \cdot \boldsymbol{T})_z &= \frac{\partial T_{zr}}{\partial r} + \frac{1}{r}\frac{\partial T_{z\theta}}{\partial \theta} + \frac{\partial T_{zz}}{\partial z} + \frac{T_{zr}}{r}.
\end{aligned}
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