# Indian Institute of Science, Bangalore

# ME 243: Midsemester Test

**Date:** 2/10/98.
**Duration:** 9.30 a.m.–11.45 a.m.
**Maximum Marks:** 100

1. Using the relation (15)
$$(\mathbf{cof}\,\boldsymbol{T})_{ij} = \frac{1}{2}\epsilon_{mni}\epsilon_{pqj}T_{mp}T_{nq},$$
prove that $\mathbf{cof}\,(\boldsymbol{RS}) = \mathbf{cof}\,(\boldsymbol{R})\mathbf{cof}\,(\boldsymbol{S})$.

2. Show that a (not necessarily symmetric) tensor $\boldsymbol{T}$ commutes with every orthogonal tensor $\boldsymbol{Q}$ if and only if $\boldsymbol{T} = \lambda\boldsymbol{I}$. (15)

3. A scalar function $\phi : V \to \Re$ is isotropic if $\phi(\boldsymbol{v}) = \phi(\boldsymbol{Qv})$ for all $\boldsymbol{v} \in V$. (20)
Show that $\phi$ is isotropic if and only if there exist a function $\hat{\phi}$ such that
$$\phi(\boldsymbol{v}) = \hat{\phi}(|\boldsymbol{v}|) \quad \forall \boldsymbol{v} \in V.$$

4. Find the gradient, $\partial\phi/\partial\boldsymbol{T}$ of the function $\phi(\boldsymbol{T}) = \det \boldsymbol{T}^{-1}$. *Derive* any results that you may need to find the final result. (15)

5. Show that the rate of deformation tensor is given by (15)
$$\boldsymbol{D} = \frac{1}{2}\boldsymbol{R}\left(\frac{D\boldsymbol{U}}{Dt}\boldsymbol{U}^{-1} + \boldsymbol{U}^{-1}\frac{D\boldsymbol{U}}{Dt}\right)\boldsymbol{R}^t,$$
where $\boldsymbol{R}$ and $\boldsymbol{U}$ are the tensors in the polar decomposition of $\boldsymbol{F}$.

6. Given the velocity field (15)
$$\boldsymbol{u} = \frac{y}{1+t^3}\boldsymbol{e}_1 + t\boldsymbol{e}_2,$$
do the following:
   (a) using the Lagrangian approach calculate the acceleration of a fluid particle having an initial position $\boldsymbol{X} = (0, 0, 0)$.
   (b) using the Eulerian definition of acceleration (i.e., the material derivative of the velocity), calculate the acceleration of the fluid.
   (c) determine the streamline that passes through the point (0,0,0).

7. Finally, a 'fun' question. The Cayley-Hamilton theorem states that a symmetric tensor satisfies its own characteristic equation. If $\lambda_1$, $\lambda_2$ and $\lambda_3$ are the eigenvalues of $\boldsymbol{S} \in$ Sym, then we can write $\boldsymbol{S}^3 - I_1\boldsymbol{S}^2 + \boldsymbol{I}_2\boldsymbol{S} - \boldsymbol{I}_3 = \boldsymbol{0}$ as $(\boldsymbol{S} - \lambda_1\boldsymbol{I})(\boldsymbol{S} - \lambda_2\boldsymbol{I})(\boldsymbol{S} - \lambda_3\boldsymbol{I}) = \boldsymbol{0}$, which in turn implies that $\boldsymbol{S} = \lambda_1\boldsymbol{I}$ or $\boldsymbol{S} = \lambda_2\boldsymbol{I}$ or $\boldsymbol{S} = \lambda_3\boldsymbol{I}$. But this is clearly not true, since $\boldsymbol{S}$ can be any symmetric tensor. Where is the catch? (5)