Indian Institute of Science, Bangalore
ME 243: Midsemester Test

Date: 27/9/11.
Duration: 7.30 a.m.–9.30 a.m.
Maximum Marks: 100

1. Is Skw a linear subspace of Lin? Does the set (12)

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \begin{bmatrix} 0 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix}, \quad \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \quad \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix},$$

constitute a basis for Skw? Justify all your answers.

2. We have seen that $\partial I_1/\partial \boldsymbol{T} = \boldsymbol{I}$ for $\boldsymbol{T} \in$ Lin. However, if $\boldsymbol{T} = \boldsymbol{W} \in$ Skw, then $I_1(\boldsymbol{W}) = 0$ for all $\boldsymbol{W}$, so that $\partial I_1/\partial \boldsymbol{W} = \boldsymbol{0}$. How do you resolve this paradox? (8)

3. Let $\{\boldsymbol{e}_1, \boldsymbol{e}_2, \ldots, \boldsymbol{e}_n\}$ be a basis for an n-dimensional vector space V. Let $\{\boldsymbol{f}_1, \boldsymbol{f}_2, \ldots, \boldsymbol{f}_m\}$, where $m > n$ be a subset of V. Is the set $\{\boldsymbol{f}_1, \boldsymbol{f}_2, \ldots, \boldsymbol{f}_m\}$ (i) linearly dependent (ii) linearly independent or (iii) cannot say, i.e., can be linearly dependent or linearly independent. *Prove* your assertion. (30)

4. For the velocity field $v_1 = \gamma x_2$, $v_2 = -\gamma x_1$, where γ is a constant, find the deformation gradient. Treat as a two-dimensional problem, and *derive* any results that you may need along the way. Your final answer should be computable using an electronic calculator, given the numerical value of γ (Hint: Find $\boldsymbol{\nabla v}$, and then proceed). (30)

5. Let $\boldsymbol{w}$ be the axial vector of $\boldsymbol{W} \in$ Skw. Find $De^{\boldsymbol{W}}(\boldsymbol{W})[\boldsymbol{U}]$ (again by deriving any results that you need), where (20)

$$e^{\boldsymbol{W}} = \boldsymbol{I} + \frac{\sin(|\boldsymbol{w}|)}{|\boldsymbol{w}|}\boldsymbol{W} + \frac{[1 - \cos(|\boldsymbol{w}|)]}{|\boldsymbol{w}|^2}\boldsymbol{W}^2.$$

Specialize to the case where $\boldsymbol{U} = \boldsymbol{S} \in$ Sym.

Some relevant formulae

$$W_{ij} = -\epsilon_{ijk} w_k,$$

$$w_i = -\frac{1}{2}\epsilon_{ijk} W_{jk},$$

$$e^{\boldsymbol{T}} = \boldsymbol{I} + \boldsymbol{T} + \frac{1}{2!}\boldsymbol{T}^2 + \cdots,$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots,$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots,$$