Indian Institute of Science, Bangalore
ME 243: Midsemester Test

Date: 29/9/12.
Duration: 9.30 a.m.–11.30 a.m.
Maximum Marks: 100

1. Let $\boldsymbol{W} \in \text{Skw}$. Determine if the set $\{\boldsymbol{I}, \boldsymbol{W}, \boldsymbol{W}^2\}$ is linearly dependent or linearly independent. (20)

2. Let $\boldsymbol{a}$ and $\boldsymbol{b}$ be two vectors . (15)

   (a) Find the axial vector of $\boldsymbol{b} \otimes \boldsymbol{a} - \boldsymbol{a} \otimes \boldsymbol{b}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$.

   (b) Using the above result, find the necessary and sufficient conditions for $\boldsymbol{b} \otimes \boldsymbol{a} = \boldsymbol{a} \otimes \boldsymbol{b}$.

3. Find a relation between (25)

$$\boldsymbol{u} \otimes (\boldsymbol{v} \times \boldsymbol{w}) + \boldsymbol{v} \otimes (\boldsymbol{w} \times \boldsymbol{u}) + \boldsymbol{w} \otimes (\boldsymbol{u} \times \boldsymbol{v})$$

   and $[\boldsymbol{u}, \boldsymbol{v}, \boldsymbol{w}]\,\boldsymbol{I}$. (Hint: Consider the tensor $\boldsymbol{T} = \boldsymbol{u} \otimes \boldsymbol{e}_1 + \boldsymbol{v} \otimes \boldsymbol{e}_2 + \boldsymbol{w} \otimes \boldsymbol{e}_3$, where $\{\boldsymbol{e}_1, \boldsymbol{e}_2, \boldsymbol{e}_3\}$ is the canonical basis). Using this relation that you have found, find a relation between $[\boldsymbol{u} \times \boldsymbol{v}, \boldsymbol{v} \times \boldsymbol{w}, \boldsymbol{w} \times \boldsymbol{u}]$ and $[\boldsymbol{u}, \boldsymbol{v}, \boldsymbol{w}]$.

4. Let $\phi(\boldsymbol{T}) = \det(e^{\boldsymbol{T}})$ and $\boldsymbol{G}(\boldsymbol{T}) = \mathbf{cof}\,\boldsymbol{T}$ for $\boldsymbol{T}$ invertible. Find $D\phi(\boldsymbol{T})[\boldsymbol{U}]$, $\partial\phi/\partial\boldsymbol{T}$ and $D\boldsymbol{G}(\boldsymbol{T})[\boldsymbol{U}]$. (20)

5. Let $\boldsymbol{F}$ denote the deformation gradient. If $\boldsymbol{T}(\boldsymbol{X})$ and $\boldsymbol{\tau}(\boldsymbol{x})$ are related by (20)

$$\boldsymbol{T}(\boldsymbol{X}) := \boldsymbol{\tau}(\boldsymbol{x})\boldsymbol{F},$$

   then find a relation between $\boldsymbol{F}[\boldsymbol{\nabla}_X \times \boldsymbol{T}(\boldsymbol{X})]$ and $\boldsymbol{\nabla}_x \times \boldsymbol{\tau}(\boldsymbol{x})$

Some relevant formulae

$$\epsilon_{pqr}(\det \boldsymbol{T}) = \epsilon_{ijk} T_{ip} T_{jq} T_{kr} = \epsilon_{ijk} T_{pi} T_{qj} T_{rk}.$$

$$\boldsymbol{W} = |\boldsymbol{w}|\,(\boldsymbol{r} \otimes \boldsymbol{q} - \boldsymbol{q} \otimes \boldsymbol{r}), \quad (\boldsymbol{w}/\,|\boldsymbol{w}|\,, \boldsymbol{q}, \boldsymbol{r} \text{ orthonormal}),$$

$$W_{ij} = -\epsilon_{ijk} w_k,$$

$$w_i = -\frac{1}{2}\epsilon_{ijk} W_{jk},$$

$$e^{\boldsymbol{T}} = \boldsymbol{I} + \boldsymbol{T} + \frac{1}{2!}\boldsymbol{T}^2 + \cdots\,,$$

$$(\boldsymbol{\nabla} \times \boldsymbol{T})_{ij} = \epsilon_{irs}\frac{\partial T_{js}}{\partial x_r}$$

$$D(\det \boldsymbol{T})[\boldsymbol{U}] = \mathbf{cof}\,\boldsymbol{T} : \boldsymbol{U},$$

$$(\mathbf{cof}\,\boldsymbol{T})_{ij} = \frac{1}{2}\epsilon_{imn}\epsilon_{jpq} T_{mp} T_{nq},$$

$$[\boldsymbol{u}, \boldsymbol{v}, \boldsymbol{w}]\,[\boldsymbol{p}, \boldsymbol{q}, \boldsymbol{r}] = \det\left[\boldsymbol{u} \otimes \boldsymbol{p} + \boldsymbol{v} \otimes \boldsymbol{q} + \boldsymbol{w} \otimes \boldsymbol{r}\right].$$