Indian Institute of Science, Bangalore
ME 243: Midsemester Test

Date: 9/10/15.
Duration: 2.30 p.m.–4.00 p.m.
Maximum Marks: 100

1. Prove that $\{\boldsymbol{u}, \boldsymbol{v}, \boldsymbol{w}\}$ is a basis for $\Re^3$ if and only if $\{\boldsymbol{u} \times \boldsymbol{v}, \boldsymbol{v} \times \boldsymbol{w}, \boldsymbol{w} \times \boldsymbol{u}\}$ is a basis for $\Re^3$. (30)

2. A continuum enthusiast tries to prove the following 'theorem': $\boldsymbol{S} \in \mathrm{Sym}$ is positive definite if and only if (20)
$$\operatorname{tr} \boldsymbol{S} > 0 \text{ and } \operatorname{tr} \boldsymbol{S}^2 > 0 \text{ and } \operatorname{tr} \boldsymbol{S}^3 > 0.$$
Determine if this result is true (Hint: The result may be false in one direction, in which case provide a counterexample.)

3. Let $\boldsymbol{Q}(\boldsymbol{W}) = (\boldsymbol{I}+\boldsymbol{W})^{-1}(\boldsymbol{I}-\boldsymbol{W})$. Determine if $\boldsymbol{Q}$ is proper orthogonal (Hint: Do $(\boldsymbol{I}+\boldsymbol{W})$ and $(\boldsymbol{I}-\boldsymbol{W})$ commute). Evaluate $D\boldsymbol{Q}(\boldsymbol{W})[\boldsymbol{U}]$. (20)

4. It is given that the rate of deformation tensor $\boldsymbol{D}$ is zero, and that the vorticity tensor is constant (i.e., not dependent on space or time). Find the velocities $\boldsymbol{v}$, $\tilde{\boldsymbol{v}}$, accelerations $\boldsymbol{a}$, $\tilde{\boldsymbol{a}}$, motion $\boldsymbol{\chi}(\boldsymbol{X}, t)$, deformation gradient $\boldsymbol{F}$ along with its polar decomposition factors $\boldsymbol{U}$, $\boldsymbol{V}$ and $\boldsymbol{R}$, the cofactor $\mathbf{cof}\,\boldsymbol{F}$, and the strain tensor $\boldsymbol{E}(\boldsymbol{X}, t)$ (all in terms of the vorticity tensor). (30)