Indian Institute of Science, Bangalore
ME 243: Midsemester Test

Date: 2/10/16.
Duration: 2.30 p.m.–4.00 p.m.
Maximum Marks: 100

1. Recall that a subset of a vector space is said to be a linear subspace if a linear combination of any two of its elements lies in it. Let (35)

$$
\begin{aligned}
V_1 &:= \{\boldsymbol{T} \in \text{Lin} : I_1(\boldsymbol{T}) = 0\}, \\
V_2 &:= \{\boldsymbol{T} \in \text{Lin} : I_2(\boldsymbol{T}) = 0\}, \\
V_3 &:= \{\boldsymbol{T} \in \text{Lin} : I_3(\boldsymbol{T}) = 0\}, \\
V_4 &:= \{\boldsymbol{T} \in \text{Lin} : I_3(\boldsymbol{T}) \neq 0\}, \\
V_5 &:= \{\boldsymbol{T} \in \text{Lin} : \|\boldsymbol{T}\| = 1\}, \\
V_6 &:= \{\mathbf{cof}\, \boldsymbol{W} : \boldsymbol{W} \in \text{Skw}\}.
\end{aligned}
$$

   Determine which of the above sets are linear subspaces of Lin. If the set is not a linear subspace, provide a counterexample (Hint for the last part: Express $\mathbf{cof}\, \boldsymbol{W}$ in terms of $\boldsymbol{w}$).

2. Show that $(\boldsymbol{u}, \boldsymbol{T}\boldsymbol{v}) = 0 \; \forall \; \boldsymbol{u}, \boldsymbol{v}$ that are mutually orthogonal, if and only if $\boldsymbol{T} = \lambda \boldsymbol{I}$. (30)

3. Starting from $\boldsymbol{n}\, dS = (\mathbf{cof}\, \boldsymbol{F})\boldsymbol{n}_0\, dS_0$, (35)

   (a) Find a relation between $\boldsymbol{n}$, $\mathbf{cof}\, \boldsymbol{F}$ and $\boldsymbol{n}_0$.

   (b) Use this relation to find the material derivative of $\boldsymbol{n}$ in terms of the velocity gradient $\boldsymbol{L}$ and $\boldsymbol{n}$ itself (Hint: You may directly express $\mathbf{cof}\, \boldsymbol{F}$ in terms of $\boldsymbol{F}^{-1}$). Derive any other results that you require on the way.

   (c) Using the results of Problem 2, find the necessary and sufficient conditions on the form of $\boldsymbol{L}$ for $D\boldsymbol{n}/Dt$ to be zero.

   (d) Assume that this form for $\boldsymbol{L}$ that you have derived in part (c) above is *independent* of $(\boldsymbol{x}, t)$. Find the velocities $\boldsymbol{v}$, $\tilde{\boldsymbol{v}}$, accelerations $\boldsymbol{a}$, $\tilde{\boldsymbol{a}}$, motion $\boldsymbol{\chi}(\boldsymbol{X}, t)$, deformation gradient $\boldsymbol{F}$ along with its polar decomposition factors $\boldsymbol{U}$, $\boldsymbol{V}$ and $\boldsymbol{R}$, and the strain tensor $\boldsymbol{E}(\boldsymbol{X}, t)$.

Some relevant formulae

$$
\begin{aligned}
W_{ij} &= -\epsilon_{ijk} w_k, \\
w_i &= -\frac{1}{2}\epsilon_{ijk} W_{jk}, \\
(\mathbf{cof}\, \boldsymbol{T})_{ij} &= \frac{1}{2}\epsilon_{imn}\epsilon_{jpq} T_{mp} T_{nq}.
\end{aligned}
$$