Indian Institute of Science, Bangalore
ME 243: Midsemester Test

Date: 28/11/20.
Duration: 2.00 p.m.–5.00 p.m.
Maximum Marks: 100

1. Let $\boldsymbol{W} \in \text{Skw}$, and let $\boldsymbol{w}$ be its axial vector. Find the dimension of $\text{Lsp}\left\{\boldsymbol{I}, \frac{\boldsymbol{W}}{|\boldsymbol{w}|}, \frac{\mathbf{cof}\,\boldsymbol{W}}{|\boldsymbol{w}|^2}\right\}$ (Hint: Find $\mathbf{cof}\,\boldsymbol{W}$ in terms of $\boldsymbol{w}$). Using this result (and without using some other method), find the dimension of $\text{Lsp}\{\boldsymbol{I}, \boldsymbol{R}, \boldsymbol{R}^T\}$ for all possible values of α, where (40)

$$\boldsymbol{R} = \boldsymbol{I} + \frac{\sin\alpha}{|\boldsymbol{w}|}\boldsymbol{W} + \frac{(1-\cos\alpha)}{|\boldsymbol{w}|^2}\boldsymbol{W}^2.$$

In case, the dimension of $\text{Lsp}\{\boldsymbol{I}, \boldsymbol{R}, \boldsymbol{R}^T\}$ for some particular value $\alpha = \alpha_0$ is different than for other values of α, find the dimension corresponding to $\alpha = \alpha_0$.

2. Consider the following problem : (25)

   (a) Starting from the equation for rigid motion $\boldsymbol{\chi} = \boldsymbol{Q}(t)\boldsymbol{X} + \boldsymbol{c}(t)$, find the expressions for the velocity and acceleration fields in the Eulerian setting.

   (b) The rigid rod AB of length L shown in Fig. 1 slides against a wall as shown. The angle $\theta(t)$ as a function of time t is *given*. You may assume the axial vector of $\dot{\boldsymbol{Q}}\boldsymbol{Q}^T$ to be given by $-\dot{\theta}(t)\boldsymbol{e}_z$ (with a superposed dot denoting a derivative with respect to time), where $\boldsymbol{e}_z = \boldsymbol{e}_x \times \boldsymbol{e}_y$. Using the results of part (a) above, find the velocity and acceleration of the center point of the rod C as a function of L, $\theta(t)$ and its derivatives with respect to time. If you make any assumptions about the accelerations of points A and B, kindly justify. You may treat this as a two-dimensional problem, i.e., the motion is in the x-y plane.

3. Let $\boldsymbol{W} \in \text{Skw}$, and let $\boldsymbol{w}$ be its axial vector. (35)

   (a) Does $\boldsymbol{W}^2 \in \text{Sym}$?

   (b) Assuming $\boldsymbol{W} = |\boldsymbol{w}|(\boldsymbol{r}\otimes\boldsymbol{q} - \boldsymbol{q}\otimes\boldsymbol{r})$, ($\boldsymbol{w}/|\boldsymbol{w}|, \boldsymbol{q}, \boldsymbol{r}$ orthonormal), and using the result of part (a), find the eigenvalues and eigenvectors of $e^{(\boldsymbol{W}^2)}$. If the eigenvectors are not unique, *choose* an orthonormal set of eigenvectors in terms of $\boldsymbol{w}$, $\boldsymbol{q}$ and $\boldsymbol{r}$, and state your choice.

   (c) Let I_i, $i = 1, 2, 3$, be the principal invariants of $e^{(\boldsymbol{W}^2)}$. Find the gradients $\partial I_i/\partial \boldsymbol{W}$, $i = 1, 2, 3$, in terms of $\boldsymbol{W}$.

Some relevant formulae

$$\boldsymbol{W} = |\boldsymbol{w}|(\boldsymbol{r}\otimes\boldsymbol{q} - \boldsymbol{q}\otimes\boldsymbol{r}), \quad (\boldsymbol{w}/|\boldsymbol{w}|, \boldsymbol{q}, \boldsymbol{r} \text{ orthonormal}),$$

$$(\mathbf{cof}\,\boldsymbol{T})^T = I_2\boldsymbol{I} - (\text{tr}\,\boldsymbol{T})\boldsymbol{T} + \boldsymbol{T}^2,$$

$$W_{ij} = -\epsilon_{ijk}w_k,$$

$$w_i = -\frac{1}{2}\epsilon_{ijk}W_{jk},$$

$$\boldsymbol{R}(\boldsymbol{w}, \alpha) = \boldsymbol{I} + \frac{1}{|\boldsymbol{w}|}\sin\alpha\,\boldsymbol{W} + \frac{1}{|\boldsymbol{w}|^2}(1-\cos\alpha)\boldsymbol{W}^2,$$

$$(\mathbf{cof}\,\boldsymbol{T})_{ij} = \frac{1}{2}\epsilon_{imn}\epsilon_{jpq}T_{mp}T_{nq},$$

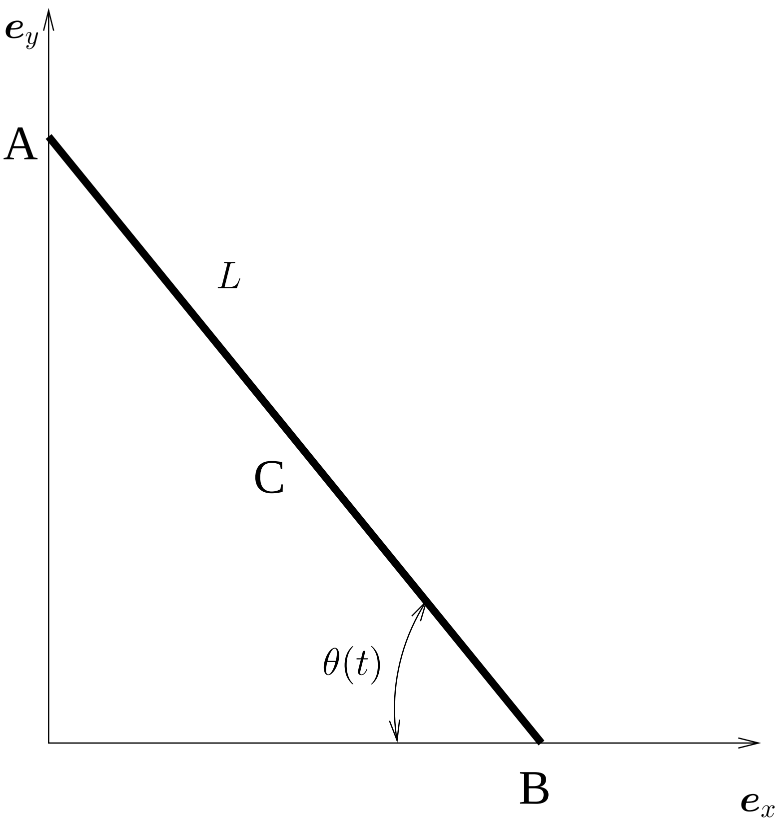

Figure 1: Rigid rod sliding against a wall.